LESSON 1 Order of Operations

☐ **BE SURE TO READ BEFORE STARTING**

- ✓ For those who need a course grade, grading sheets are available in Appendix A. Record your score after completing each lesson. See the grading sheets for details.
- ✓ It is a good practice to solve the problems in a separate notebook. Start each lesson on a fresh page. Write the lesson number on the top of the page and label each problem with its number. This will help you when you are checking the answers and when you review your work later.
- ✓ This workbook tries to provide ample practice problems. You will find problems marked "**EXTRA**" and "**CHALLENGE**." They are optional and are not required assignments. Use extra problems for more practice, especially if you are struggling, or save them for later review.

☐ **BASIC DEFINITIONS**

An **expression** is a mathematical statement that does not contain an equal sign. **Evaluating an expression** means finding its value.

An **exponent** is a way to represent how many times a number is multiplied by itself. The expression b^n means the number b is multiplied n times. The repeated number b is called the **base**, and the number n is called the **exponent** or **power**. For example, 2^3 is the same as saying $2 \times 2 \times 2$. In words, 2^3 is read as "two to the third power" or "two cubed."

☐ **EVALUATING EXPONENTS**

To evaluate b^n, simply multiply b by itself n times.

Note that $(-5)^2$ and -5^2 evaluate to different values. The expression $(-5)^2$ represents $(-5) \times (-5)$ which evaluates to 25. The expression -5^2 represents $-(5 \times 5)$ which evaluates to -25.

➔ **EXAMPLE** Evaluate 2^3 and $(-5)^2$.

$2^3 = 2 \times 2 \times 2 = 8$

$(-5)^2 = (-5) \times (-5) = 25$

➔ **TRY IT** Evaluate.

1. 3^4

2. $(-10)^3$

☐ **USING THE ORDER OF OPERATIONS**

When evaluating expressions, remember to follow the **order of operations**. To get correct answers, you must perform operations in the proper order: Parentheses, Exponents, Multiplication and Division in order from left to right, then Addition and Subtraction in order from left to right. These rules are often referred to as **PEMDAS** (or **P**lease **E**xcuse **M**y **D**ear **A**unt **S**ally).

➔ **EXAMPLE** Evaluate $9 + 3 \times 3 - 5$.

$9 + 3 \times 3 - 5$	Multiplication
$= 9 + 9 - 5$	Addition, then Subtraction (left to right)
$= 18 - 5$	
$= 13$	

➔ **TRY IT** Evaluate.

3. $5 - 8 \times 4 + 2$

4. $3 \times 8 \div 6 - 2 + 7$

➔ **EXAMPLE** Evaluate $10 \div 2 \times (4 - 6)$.

$10 \div 2 \times (4 - 6)$	Parentheses
$= 10 \div 2 \times (-2)$	Division, then Multiplication (left to right)
$= 5 \times (-2)$	
$= -10$	

➔ **TRY IT** Evaluate.

5. $15 - 8 \times (3 + 2)$

6. $3 \times (6 \div 6) - 2 + 7$

➔ **EXAMPLE** Evaluate $4 - 8 \div (5 - 3)^2 \times 5$.

$4 - 8 \div (5 - 3)^2 \times 5$	Parentheses
$= 4 - 8 \div 2^2 \times 5$	Exponents
$= 4 - 8 \div 4 \times 5$	Division
$= 4 - 2 \times 5$	Multiplication
$= 4 - 10$	Subtraction
$= -6$	

➔ **TRY IT** Evaluate.

7. $1^4 - 6 \times (3 \div 3)^6$

8. $5 + 2^5 \div 8 \times (2 - 4)^2 - 3$

☐ **EXERCISE YOUR SKILLS**

Evaluate.

9. 1^8 **10.** $(-2)^5$ **11.** -3^3 **12.** $(-9)^2$

Evaluate using the order of operations.

13. $7 - 3 + 2$

14. $12 \div 3 \times 2$

15. $-34 + 3 \times 5$

16. $4 \times (5 + 6)$

17. $3^2 + 6 \times (-7)$

18. $(6 + 2)^2 \div (-4)^2$

19. $9 + 5 \times (8 \div 8)^5$

20. $(7 - 3) \div 2^2 \times (-2)^4$

21. $30 - 8^2 \div 2^5 - 5^2$

22. $4 + 3^3 \div 9 \times (6 - 7)^2$

CHALLENGE Evaluate using the order of operations.

23. $9 + (3 \times 2^3 - 8 \div 2) - 4^2$

24. $36 \div (6^2 - 7 \times 5)^4 + 7$

25. $\dfrac{(-4) \times (5 + 2) + 3}{3^2 - 2^4 \div 4}$

26. $\dfrac{5 - 4^2 \div 8 \times (3 - 5)^3}{(3^2 - 7) \times 2 + (-3)^1}$

LESSON 9 Applications of Linear Equations

☐ SOLVING LINEAR EQUATION WORD PROBLEMS

There are many types of word problems that require algebra: number problems, geometry problems, money problems, percentage problems, distance problems, mixture problems, and so on. You may find some more difficult than others, but they all can be solved using the same strategy. To solve a word problem using algebra, 1) define a variable, 2) set up an equation to model the given situation, 3) solve the equation as usual, and then 4) answer what's being asked.

➔ **EXAMPLE** Consecutive integers

The sum of three consecutive even integers is 12. Find the integers.

1. Let x = the first even integer
2. $x + 2$ = the second even integer
   $x + 4$ = the third even integer
   The sum is 12, so $x + (x + 2) + (x + 4) = 12$.
3. Solve for x, and you get $x = 2$.
4. The numbers are 2, 4, and 6.

➔ **TRY IT** Solve.

1. The sum of two consecutive even integers is 26. Find the two integers.

2. The sum of two consecutive odd integers is 32. Find the two integers.

3. The sum of three consecutive integers is 27. Find the integers.

➔ **EXAMPLE** Percent

The price of an apple rose by 15% to $1.38/lb. What was the original price?

1. Let x = the original price
2. $0.15x$ = the price increase
   The new price = the original price + the price increase, so $1.38 = x + 0.15x$.
3. Solve for x, and you get $x = 1.2$.
4. The original price was $1.20/lb.

➔ **TRY IT** Solve.

4. Cammy bought a jacket at $43.50. The price was 25% off the regular price. What was the regular price?

5. Jim bought a pair of pants at $16.80. The price was 40% off the regular price. What was the regular price?

➔ **EXAMPLE** Age

Dale is 6 years older than Kate. Three years ago, Dale was twice as old as Kate. How old are they now?

1. Let x = Kate's age now
2. $x + 6$ = Dale's age now
   $x - 3$ = Kate's age 3 years ago
   $(x + 6) - 3$ = Dale's age 3 years ago
   Dale's age 3 years ago = twice Kate's age 3 years ago, so $(x + 6) - 3 = 2(x - 3)$.
3. Solve for x, and you get $x = 9$.
4. Kate is 9 years old. Dale is 15 years old.

➔ **TRY IT** Solve.

6. Jamie is 5 years older than Nicole. Two years ago, Jamie was twice as old as Nicole. How old are they now?

7. Mia's father is 42 years old. Six years ago, he was six times as old as Mia. How old is Mia?

8. Ellen is 10 years younger than Max. In two years, Max will be twice as old as Ellen. How old are they now?

➔ **EXAMPLE** Geometry

The length of a rectangle is twice its width. The perimeter is 18 feet. Find the dimensions of the rectangle.

1. Let x = the width of the rectangle
2. $2x$ = the length of the rectangle
   Perimeter = 2(length + width),
   so $2(x + 2x) = 18$.
3. Solve for x, and you get $x = 3$.
4. The rectangle is 3 feet by 6 feet.

➔ **TRY IT** Solve.

9. The length of a rectangle is three times its width. The perimeter is 40 feet. Find the dimensions of the rectangle.

10. The length of a rectangle is 5 cm less than three times its width. The perimeter is 22 cm. Find the dimensions of the rectangle.

➔ **EXAMPLE** Coins

Olivia has $0.90 in dimes and nickels. She has three more nickels than dimes. How many coins of each type does she have?

1. Let x = the number of dimes
2. $x + 3$ = the number of nickels
   Total value = x dimes at $0.10 each + $(x + 3)$ nickels at $0.05 each,
   so $0.10x + 0.05(x + 3) = 0.90$.
3. Solve for x, and you get $x = 5$.
4. Olivia has 5 dimes and 8 nickels.

➔ **TRY IT** Solve.

11. Emma has $0.95 in dimes and nickels. She has five more dimes than nickels. How many coins of each type does she have?

12. Joey has $1.60 in quarters and nickels. He has two more nickels than quarters. How many coins of each type does he have?

☐ **EXERCISE YOUR SKILLS**

For each problem, 1) define a variable, 2) set up an equation, 3) solve the equation, and 4) answer what's being asked. Show your work in your notebook.

13. One integer is 5 less than twice another. Their sum is 25. Find the two integers.

14. The sum of three consecutive odd integers is 27. Find the integers.

15. A company's stock price dropped by 10% to $108 per share. What was the previous price per share?

16. Currently, Joey is three times as old as Anna. In five years, Joey will be twice as old as Anna. How old is Joey? How old is Anna?

17. Two sides of a triangle are equal in length and twice the length of the shortest side. The perimeter is 45 inches. Find the dimensions of the triangle.

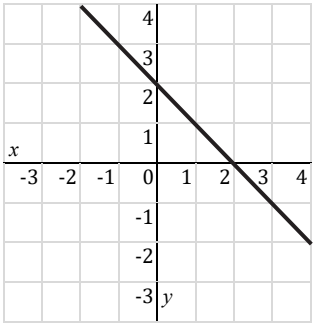
18. Max has one-, five-, and ten-dollar bills totaling $82. He has twice as many fives as ones and three times as many tens as ones. How many bills of each type does he have?

LESSON 28 Catch Up and Review!

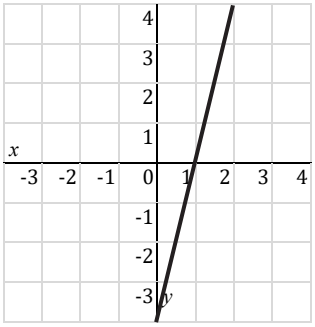
Catch up if you are behind. Use the review problems below to make sure you're on track.

LESSON 22 Find an equation of each line in slope-intercept form.

1.



2.



LESSON 23 Find an equation of each line in slope-intercept form.

3. slope $= 2$; through $(0, 4)$

4. slope $= 5$; through $(2, 9)$

5. through $(4, 9)$ and $(-1, -6)$

6. through $(3, 2)$ and $(4, 0)$

LESSON 24 Find an equation of each line in point-slope form. Use the first point for point-slope form when given two points.

7. slope $= -1$; through $(3, 0)$

8. slope $= 2$; through $(5, 4)$

9. through $(-2, -7)$ and $(1, 5)$

10. through $(3, -8)$ and $(2, -3)$

LESSON 25 Find an equation of each line in standard form. Use only integers and the smallest possible positive integer coefficient for x.

11. slope $= 5$; through $(2, 4)$

12. slope $= \frac{4}{5}$; through $(-5, 3)$

13. through $(5, 3)$ and $(-5, 1)$

14. through $(-4, 8)$ and $(6, 3)$

LESSON 26 Solve.

15. A taxi charges a flat fee of $5 and $1.60 per mile.

a. Write an equation representing the total cost, y, of riding the taxi for x miles.

b. How much will a taxi ride cost for 15 miles?

c. If a taxi ride cost $45, how many miles did the taxi travel?

16. An internet service provider charges $32 per month plus an initial set-up fee of $58.

a. Write an equation representing the total cost, y, after x months of service.

b. How much will it cost after 5 months of service?

c. If a customer spent a total of $442, how long was the service provided?

LESSON 27 Solve.

17. Mia has x quarters and y dimes amounting to $2.30.

a. Write an equation relating x and y.

b. If she has 6 quarters, how many dimes does she have?

18. At a grocery, Rodney bought 3 bags of onions and 2 bags of potatoes. He spent $20 in total. Onions cost x per bag, and potatoes cost y per bag.

a. Write an equation relating x and y.

b. If each bag of potatoes costs $5.50, how much does each bag of onions cost?

☐ **MIXED REVIEW: PRE-ALGEBRA**

Brush up on the topics covered in Pre-Algebra.

19. What is 20% of 80?

20. Convert 40% to a fraction. Simplify your answer.

21. You cut 17 feet from a 50-foot wire. What is the percent decrease in length?

22. One kilometer is 1000 meters. One meter is 100 centimeters. How many centimeters are there in one kilometer?

23. One meter is approximately 3 feet. How many centimeters are there in 1.2 feet?

LESSON 46 PSAT Practice

This is a timed practice test. Get a timer, a bubble answer sheet (provided in Appendix B), and blank sheets of paper for your calculations. When you are ready, set the timer for **25 minutes** and begin. Do not use a calculator. Mark all your answers on the answer sheet. Only answers marked on the answer sheet can be scored. After the test, make sure you review what you missed.

1. Which of the following is equivalent to the expression $4 - 3(x - 2) + 2x$?

A) $-x - 2$ B) $-x + 10$

C) $5x - 2$ D) $5x + 10$

2. $2x - 5 = 4 - x$

If x is the solution to the equation above, what is the value of $x + 3$?

A) -3 B) 0 C) 3 D) 6

3. $2x + y = 2$ and $x + 3y = -9$

Which ordered pair (x, y) satisfies the system of equations above?

A) $(-2, 6)$ B) $(0, -3)$

C) $(1, 0)$ D) $(3, -4)$

4. Which of the following is equivalent to the inequality $14 - 2x > 3(x - 2)$?

A) $x > -4$ B) $x > 4$

C) $x < 4$ D) $x > 8$

5. $7 + 3x = 3(x + c) - 5$

Which value of c makes the equation above have infinitely many solutions?

A) 0 B) 1 C) 2 D) 4

6. $|3 - 2x| = 7$

If a and b are the solutions to the equation above, what is the value of $a + b$?

A) 3 B) 5 C) 7 D) 9

7. $x - y = 5$ and $x + 2y = -1$

If (p, q) is a solution to the system above, what is the value of p ?

A) -1 B) -2 C) 3 D) 6

8. $F = \frac{9}{5}C + 32$

The formula above gives the Fahrenheit temperature F for a given Celsius temperature C. Which formula gives the Celsius temperature C for a given Fahrenheit temperature F ?

A) $C = \frac{5}{9}F - 32$ B) $C = \frac{5}{9}(F - 32)$

C) $C = \frac{9}{5}F - 32$ D) $C = \frac{9}{5}(F - 32)$

9. A recipe calls for 2 quarts of milk, but Josh has only 2 cups of milk. How much more milk does he need in cups? (1 quart = 2 pints and 1 pint = 2 cups)

A) 2 B) 4 C) 6 D) 8

Continue to the next page.

10. A hiking club has 45 members. The ratio of males to females is 2:3. How many males are in the club?

A) 9 B) 18 C) 27 D) 30

11. The line with the equation $x - y = 3$ does NOT pass through which of the four quadrants?

A) I B) II C) III D) IV

12. $y = 50 + 30x$

The equation above models the total cost, y, that an electrician charges for x hours of service. The total cost consists of a one-time fee plus an hourly charge. If the equation is graphed in the *xy*-plane, what is indicated by the *y*-intercept of the graph?

A) A one-time fee of \$30

B) A one-time fee of \$50

C) An hourly charge of \$30

D) An hourly charge of \$50

13. An airplane 8 kilometers above the ground begins descending at an average speed of 350 meters per minute. Which expression represents the altitude of the plane, in kilometers, after t minutes?

A) $8 - 0.35t$ B) $8 - 350t$

C) $8000 - 0.35t$ D) $8000 - 350t$

14. Mark is 18 years old now. Two years ago, Mark was twice as old as Kate. How many years older than Kate is Mark?

A) 6 B) 8 C) 10 D) 12

15. Natalie bought a hat using a \$2 coupon off the regular price. With sales tax of 5% added, she paid \$8.40 in total. Which equation can be used to determine the regular price, x, of the hat?

A) $1.05x + 2 = 8.4$

B) $1.05x - 2 = 8.4$

C) $1.05(x + 2) = 8.4$

D) $1.05(x - 2) = 8.4$

16. A group of x adults and y children went to see a movie. Movie tickets cost \$8 for adults and \$6 for children. The group bought 10 tickets and paid \$72 in total. Which system of equations represents the relationship between x and y ?

A) $x + y = 10$ and $8x + 6y = 72$

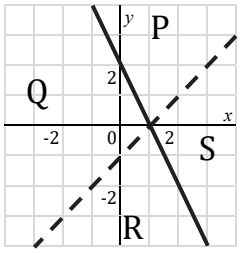
B) $x + y = 10$ and $8x + 6y = 10 \cdot 72$

C) $x + y = 72$ and $8x + 6y = 10$

D) $x + y = 72$ and $8x + 6y = 10 \cdot 72$

17. $x - y > 1$ and $2x + y \leq 2$

The system of inequalities above is graphed below. Which region represents the solution to the system?



A) Region P

B) Region Q

C) Region R

D) Region S

STOP

This is the end of the test. If you finish before time is up, check your work.

LESSON 104 Solving Quadratics by Taking Square Roots

☐ **REFRESH YOUR SKILLS**

Solve. Review Lessons 3 and 103 if needed.

1. $2x + 3 = 9$

2. $2x^2 - 9 = 7$

☐ **SOLVING QUADRATICS BY TAKING SQUARE ROOTS**

You can also solve a quadratic equation of the form $a(px + q)^2 + r = 0$ using the square root property. The key is to isolate the squared term first.

➔ **EXAMPLE** Solve by taking square roots.

$(x - 5)^2 - 9 = 0$

$(x - 5)^2 = 9$ Isolate the squared term.

$x - 5 = \pm\sqrt{9}$ Apply the square root property.

$x - 5 = \pm 3$ Simplify the radical.

$x = 5 \pm 3$ Solve for x.

$x = 8, x = 2$ Separate the solutions.

➔ **TRY IT** Solve.

3. $(x - 9)^2 = 4$

4. $(x + 5)^2 - 81 = 0$

➔ **EXAMPLE** Solve by taking square roots.

$(2x - 8)^2 - 24 = 0$

$(2x - 8)^2 = 24$ Isolate the squared term.

$2x - 8 = \pm\sqrt{24}$ Apply the square root property.

$2x - 8 = \pm 2\sqrt{6}$ Simplify the radical.

$2x = 8 \pm 2\sqrt{6}$

$x = 4 \pm \sqrt{6}$ Solve for x.

➔ **TRY IT** Solve.

5. $(3x - 5)^2 = 16$

6. $(4x + 12)^2 - 32 = 0$

➔ **EXAMPLE** Solve by taking square roots.

$2(3x + 1)^2 - 5 = 7$

$2(3x + 1)^2 = 12$

$(3x + 1)^2 = 6$ Isolate the squared term.

$3x + 1 = \pm\sqrt{6}$ Apply the square root property.

$x = \dfrac{-1 \pm \sqrt{6}}{3}$ Solve for x.

➔ **TRY IT** Solve.

7. $2(5x - 1)^2 = 20$

8. $5(4x + 3)^2 - 10 = 0$

As shown in the examples above, the plus or minus symbol is often used to condense the two solutions into one. You could separate the solutions and list them individually.

☐ **EXERCISE YOUR SKILLS**

Solve by taking square roots. Check your solutions.

9. $(x+5)^2 = 90$

10. $(x-1)^2 - 5 = 0$

11. $(x-7)^2 + 5 = 54$

12. $(x+1)^2 - 78 = 72$

13. $3(x-1)^2 = 75$

14. $(4x-3)^2 = 25$

15. $3(x-4)^2 - 7 = 20$

16. $5(x+2)^2 - 12 = 53$

17. $(2x+3)^2 - 6 = 0$

18. $(5x-2)^2 - 45 = 0$

19. $2(3x-1)^2 - 16 = 0$

20. $6(3x+5)^2 - 96 = 0$

EXTRA Solve by taking square roots. Check your solutions.

21. $x^2 + 4 = 16$

22. $3x^2 - 27 = 0$

23. $(x-3)^2 = 20$

24. $(9x-5)^2 = 0$

25. $4(x+1)^2 = 36$

26. $(x-4)^2 - 48 = 0$

27. $(2x+6)^2 - 28 = 0$

28. $3(x+4)^2 - 12 = 78$

29. $4(x-7)^2 - 10 = 54$

30. $2(4x+1)^2 - 24 = 0$

CHALLENGE Solve. (*Hint*: Factor the left side.)

31. $x^2 + 2x + 1 = 5$

32. $x^2 - 6x + 9 = 7$

33. $x^2 - 10x + 25 = 9$

34. $x^2 + 12x + 36 = 8$

35. $4x^2 + 4x + 1 = 49$

36. $4x^2 - 12x + 9 = 25$