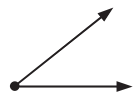
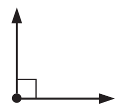
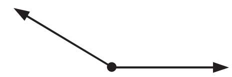
Lesson 8 Angle Types and Pairs

☐ CLASSIFYING ANGLES

Angles can be classified by their measures. **Acute** angles are less than 90°. **Right** angles measure exactly 90°. **Obtuse** angles are between 90° and 180°. **Straight** angles are exactly 180°.


Acute angle


Right angle


Obtuse angle

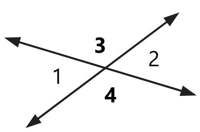
Straight angle

➔ **TRY IT 1.** An angle measures 125°. Classify it as *acute*, *right*, *obtuse*, or *straight*.

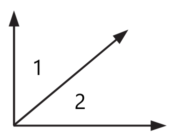
➔ **TRY IT 2.** True or false? Two right angles can form a straight angle.

☐ IDENTIFYING ANGLE PAIRS FORMED BY INTERSECTING LINES

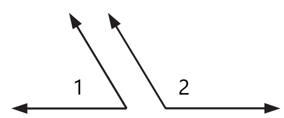
There are pairs of angles with special names. **Vertical angles** are angles opposite each other where two lines cross. **Complementary angles** are two angles whose sum is 90°. **Supplementary angles** are two angles whose sum is 180°. **Adjacent angles** are two angles that have a common vertex and a common side but do not overlap. Complementary and supplementary angles do not have to be adjacent. A pair of adjacent angles that form a straight angle is called a **linear pair**.



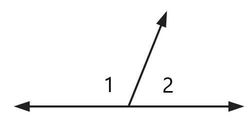
Vertical angles



Complementary adjacent angles

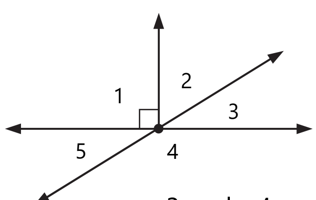


Supplementary non-adjacent angles



Linear pair

➔ **EXAMPLE** Name a linear pair.



∠3 and ∠4 are a linear pair.

➔ **TRY IT** Use the diagram on the left.

3. Name another linear pair.

4. Name a pair of vertical angles.

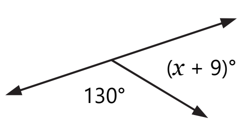
5. Name a pair of complementary angles.

6. Name all angles adjacent to an obtuse angle.

☐ FINDING ANGLES BETWEEN INTERSECTING LINES

Now you can use angle pair relationships to find angle measures.

➔ **EXAMPLE** Find the value of x.



The two angles are supplementary.
$130 + (x + 9) = 180$
Solve for x, and $x = 41$.

➔ **TRY IT** Use the diagram in the previous example.

7. Find each angle measure when $m\angle 2 = 57°$.

8. Find each angle measure when $m\angle 3 = 35°$.

9. Find $m\angle 3$ and $m\angle 5$ when $m\angle 4 = 150°$.

10. True or false? $\angle 3$ and $\angle 5$ are always congruent.

☐ EXERCISE YOUR SKILLS

Name each angle in four ways. Then classify the angle as *acute*, *right*, *obtuse*, or *straight*.

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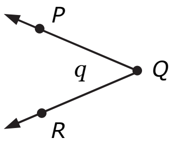

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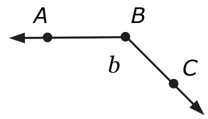

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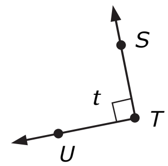

Classify each pair of angles as *vertical*, *complementary*, *supplementary*, or none of these.

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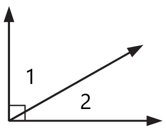

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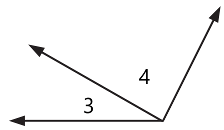

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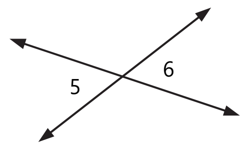

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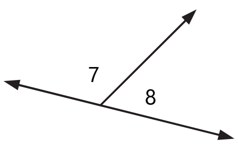

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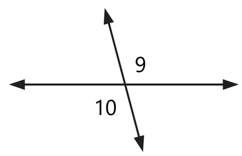

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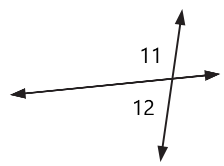

Find the value of x.

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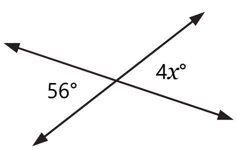

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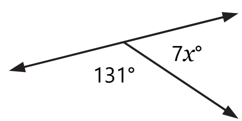

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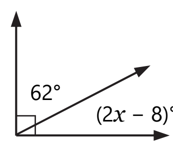

(HONORS) Solve.

23. True or false? Angles in a linear pair are complementary.

24. True or false? The supplement of an acute angle is obtuse.

25. True or false? Complementary angles are both acute angles.

26. True or false? Two obtuse angles can be supplementary to each other.

27. Two complementary angles are congruent. What is the measure of each angle?

28. Two congruent angles form a linear pair. What is the measure of each angle?

29. Two vertical angles are complementary. What is the measure of each angle?

30. An angle is the supplement of a right angle. What is the measure of the angle?

☐ MIXED REVIEW

31. (Lesson 1) Simplify $4\sqrt{12} - \sqrt{27}$.

32. (Lesson 7) Points A, B, and C are collinear. Point B is between points A and C. What is the length of $\overline{AB}$ if $AB = x$, $BC = 2x$, and $AC = 18$. (*Hint*: Use the Segment Addition Postulate.)

LESSON 40 Isosceles and Equilateral Triangles

☐ **REFRESH YOUR SKILLS**

(Lesson 11) Find the value of x.

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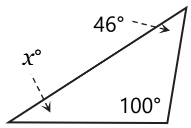

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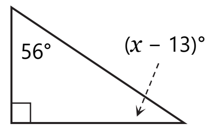

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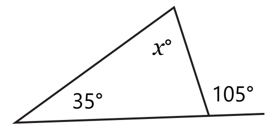
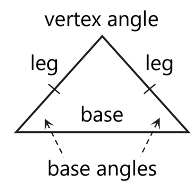
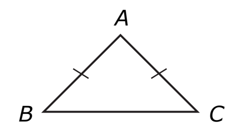


☐ **UNDERSTANDING ISOSCELES AND EQUILATERAL TRIANGLES**

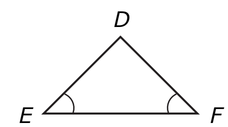
An **isosceles triangle** is a triangle with at least two congruent sides. The two congruent sides of an isosceles triangle are called the **legs**. The angle formed by the legs is called the **vertex angle** or **apex angle**. The side opposite the vertex angle is called the **base**. The two angles adjacent to the base are called the **base angles**. Here are two important theorems about isosceles triangles.



Theorem 40.1 Base Angles Theorem (Proof: #5)
If two sides of a triangle are congruent, then the angles opposite the sides are congruent.



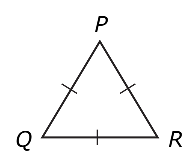
Theorem 40.2 Base Angles Converse (Proof: #6)
If two angles of a triangle are congruent, then the sides opposite the angles are congruent.

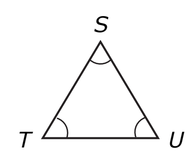


The following theorems follow immediately from Theorem 40.1 and Theorem 40.2.

Theorem 40.3 Equilateral Triangle Theorem (Proof: #21)
If a triangle is equilateral, then it is equiangular.

Theorem 40.4 Equilateral Triangle Converse (Proof: #22)
If a triangle is equiangular, then it is equilateral.

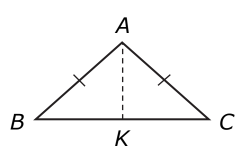




➔ **TRY IT 4.** Restate each theorem in symbols using the diagrams above.

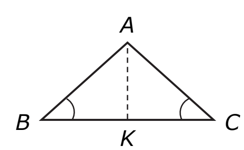
➔ **TRY IT** Complete the proofs of Theorems 40.1 and 40.2.

5. Given: $\overline{AB} \cong \overline{AC}$
Prove: $\angle B \cong \angle C$



STATEMENTS	REASONS
S 1. $\overline{AB} \cong \overline{AC}$	1. Given
2. Draw $\overline{AK}$ bisecting $\angle A$.	2. Construction
A 3. $\angle BAK \cong \angle CAK$	3.
S 4. $\overline{AK} \cong \overline{AK}$	4.
5. $\triangle ABK \cong \triangle ACK$	5.
6. $\angle B \cong \angle C$	6.

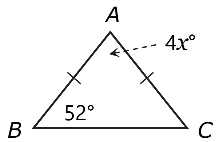
6. Given: $\angle B \cong \angle C$
Prove: $\overline{AB} \cong \overline{AC}$



STATEMENTS	REASONS
A 1. $\angle B \cong \angle C$	1. Given
2. Draw $\overline{AK}$ bisecting $\angle A$.	2. Construction
A 3. $\angle BAK \cong \angle CAK$	3.
S 4. $\overline{AK} \cong \overline{AK}$	4.
5. $\triangle ABK \cong \triangle ACK$	5.
6. $\overline{AB} \cong \overline{AC}$	6.

☐ SOLVING PROBLEMS INVOLVING ISOSCELES AND EQUILATERAL TRIANGLES

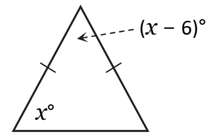
➔ **EXAMPLE** Find the value of x.



By Thm 40.1, $m\angle B = m\angle C$.
$m\angle A + m\angle B + m\angle C = 180°$
$4x + 52 + 52 = 180$
Solve for x to get $x = 19$.

➔ **TRY IT** Find the value of x.

7.



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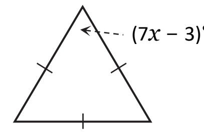
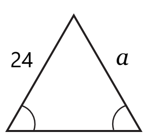


☐ EXERCISE YOUR SKILLS

Find the values of the variables.

9.



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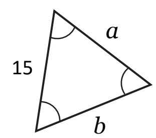

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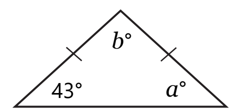

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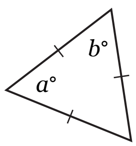

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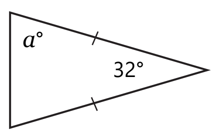

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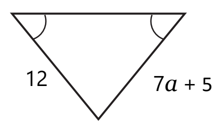

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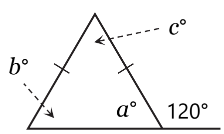

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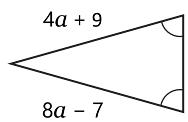

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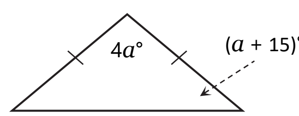
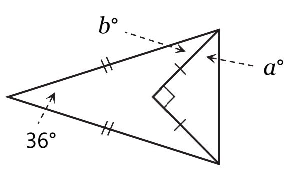


(HONORS) Find the values of the variables.

18.



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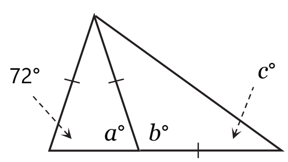

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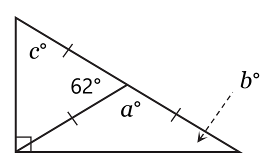
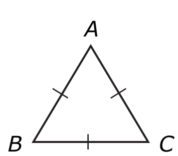
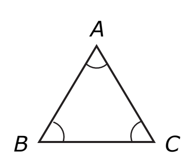


(HONORS) The proofs of Theorems 40.3 and 40.4 are straightforward when you use Theorems 40.1 and 40.2. Write a two-column proof of each theorem.

21. Given: equilateral $\triangle ABC$
Prove: $\triangle ABC$ is equiangular.



22. Given: equiangular $\triangle ABC$
Prove: $\triangle ABC$ is equilateral.



☐ MIXED REVIEW

23. (Lessons 8 & 9) All (right, vertical, adjacent, straight, complementary, corresponding) angles are congruent. Which word makes the sentence correct? Select all that apply.

24. (Lesson 35) $\triangle ABC \cong \triangle DEF$, $m\angle A = 45°$, and $m\angle E = 80°$. What is $m\angle C$?

LESSON 61 Catch Up and Review!

Catch up if you are behind. Use the review problems below to make sure you're on track.

(Lesson 56) Find the values of the variables in each parallelogram.

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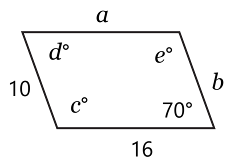

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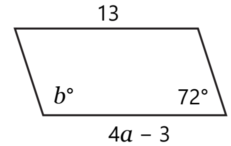

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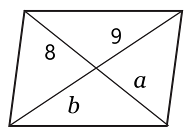

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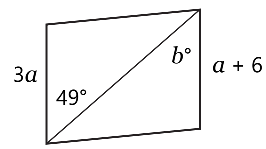

(Lesson 57) Determine if each quadrilateral is a parallelogram. Explain.

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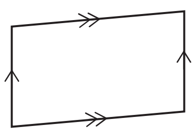

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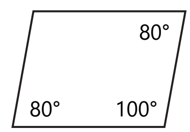

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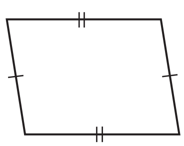

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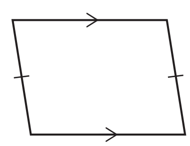

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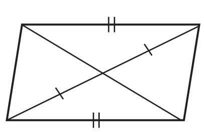

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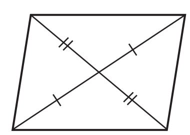
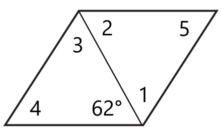


(Lesson 58) Find the measures of the numbered angles in each rhombus.

11.



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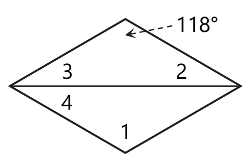

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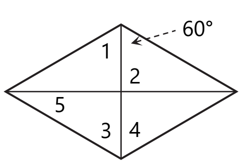

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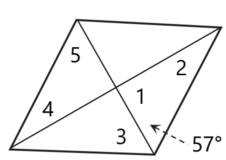
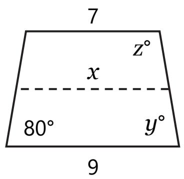


(Lesson 59) Find the values of the variables in each isosceles trapezoid with a midsegment.

15.



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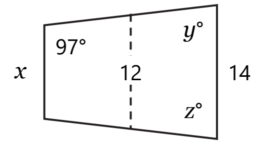
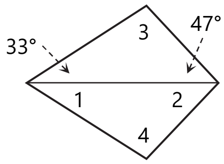
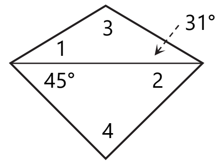


(Lesson 59) Find the measures of the numbered angles in each kite.

17.

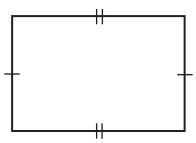


18.



(Lessons 56 ~ 59) State the most specific name for each quadrilateral.

19.



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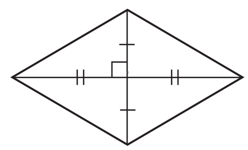

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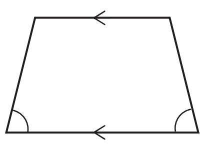

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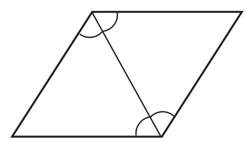
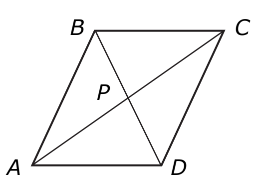


(Lesson 56) Complete the proof of the Parallelogram Diagonals Theorem [56.4].

23. Given: $\square ABCD$

Prove: $\overline{PA} \cong \overline{PC}, \overline{PB} \cong \overline{PD}$



STATEMENTS	REASONS
1. $\square ABCD$	1. Given
2. $\overline{BA} \parallel \overline{CD}, \overline{BC} \parallel \overline{AD}$	2.
3. $\angle PAB \cong \angle PCD, \angle PBA \cong \angle PDC$	3.
4. $\overline{AB} \cong \overline{CD}$	4.
5. $\triangle PAB \cong \triangle PCD$	5.
6. $\overline{PA} \cong \overline{PC}, \overline{PB} \cong \overline{PD}$	6.

☐ **MIXED REVIEW**

24. (Lesson 33) A line is reflected over its perpendicular line. What is the image of the line?

25. (Lesson 35) If $\triangle ABC \cong \triangle ACB$, what can you conclude about $\triangle ABC$?

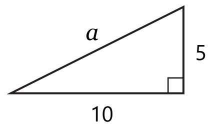
26. (Lesson 46) In $\triangle ABC$, D, E, and F are the midpoints of the sides respectively. What is the relationship between the perimeter of $\triangle ABC$ and the perimeter of $\triangle DEF$?

LESSON 103 Areas of Triangles and Quadrilaterals

☐ REFRESH YOUR SKILLS

(Lessons 74 & 76) Find the values of the variables in simplest radical form.

1.



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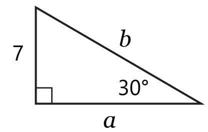
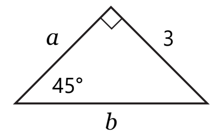


3.



(Lessons 56 ~ 59) Name all quadrilaterals that have each property.

4. Opposite sides are congruent.

5. Diagonals bisect each other.

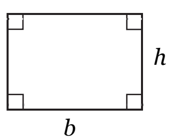
6. Diagonals are perpendicular.

7. Diagonals are congruent.

☐ DERIVING AREA FORMULAS FOR TRIANGLES AND QUADRILATERALS

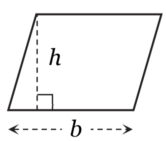
Recall from Pre-Algebra that **area** is the number of unit squares needed to fill a flat figure. A unit square is a square with a side length of 1, and it has an area of 1. Area is measured in square units. Below are the area formulas for common figures. You should be familiar with the first four.

Rectangle



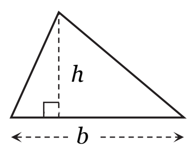
$A = bh$

Parallelogram



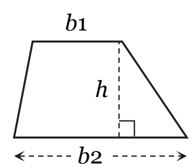
$A = bh$

Triangle



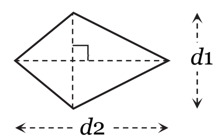
$A = \frac{1}{2}bh$

Trapezoid



$A = \frac{1}{2}h(b_1 + b_2)$

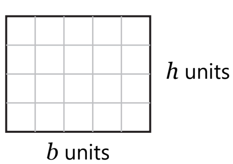
Rhombus/Kite



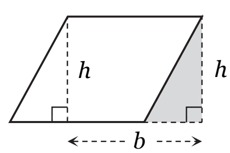
$A = \frac{1}{2}d_1 d_2$

The diagrams below show how these formulas are all derived from the area formula of a rectangle.

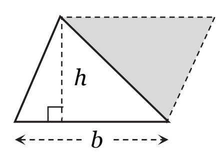
Area of the rectangle
= Number of unit squares
= b units × h units



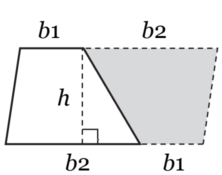
Area of the parallelogram
= Area of the rectangle
= bh



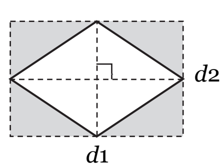
Area of the triangle
= Half the parallelogram
= Half of bh



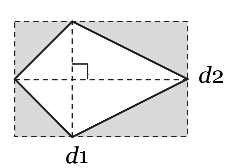
Area of the trapezoid
= Half the parallelogram
= Half of $(b_1 + b_2)h$



Area of the rhombus
= Half the large rectangle
= Half of $d_1 d_2$



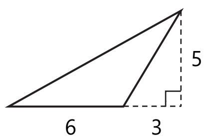
Area of the kite
= Half the large rectangle
= Half of $d_1 d_2$



☐ FINDING AREAS OF TRIANGLES AND QUADRILATERALS

When calculating the area, 1) be sure to use the base *perpendicular* to the given altitude, 2) ignore extraneous information, and 3) note that you may need to find missing dimensions first.

➔ EXAMPLE Find the area of the triangle.



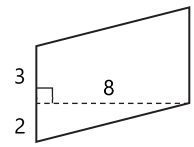
Notice that b = 6, not 3.

$A = \frac{1}{2}bh = \frac{1}{2}(6)(5)$

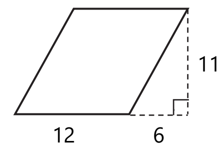
$= 15$ square units

➔ TRY IT Find the area of each parallelogram.

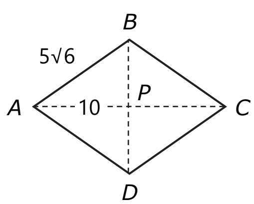
8.



9.



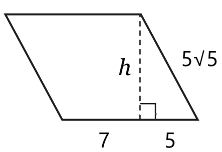
➔ EXAMPLE $ABCD$ is a rhombus. $AB = 5\sqrt{6}$ and $AP = 10$. Find the area.



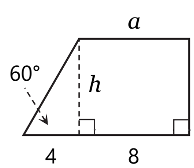
1. The diagonals of a rhombus are perpendicular, so $\triangle ABP$ is a right triangle. Use the Pythagorean Theorem to get $BP = \sqrt{AB^2 - AP^2} = 5\sqrt{2}$.
2. A rhombus is a parallelogram and the diagonals of a parallelogram bisect each other, so $d1 = AC = 2AP = 20$ and $d2 = BD = 2BP = 10\sqrt{2}$.
3. So, the area is $A = \frac{1}{2}(d1)(d2) = \frac{1}{2}(20)(10\sqrt{2}) = 100\sqrt{2}$ square units.

➔ TRY IT Find the values of the variables, then find the area in simplest radical form.

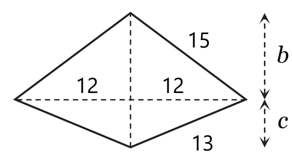
10. parallelogram



11 trapezoid



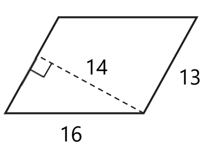
12. kite



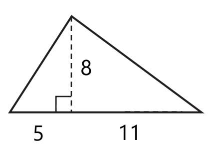
☐ EXERCISE YOUR SKILLS

Find the values of the variables, if any, then find the area in simplest radical form.

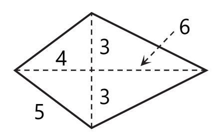
13. parallelogram



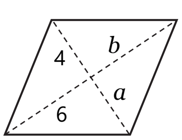
14. triangle



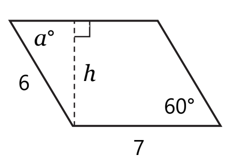
15. kite



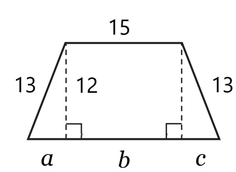
16. rhombus



17. parallelogram



18. trapezoid



☐ MIXED REVIEW

19. (Lesson 78) A right triangle has acute angles $a°$ and $b°$. If $\sin a° = 4/5$, what is $\cos b°$?

20. (Lesson 96) A right triangle is inscribed in a circle of diameter 8 cm. One of the angles of the triangle is 60°. What is the perimeter of the triangle in simplest radical form?

Lesson 165 Review: Similar Triangles

1. Which statement is true about similar polygons? Select all that apply.

A) Corresponding sides are congruent.

B) Corresponding angles are congruent.

C) Corresponding sides are proportional.

D) Corresponding angles are proportional.

2. Which statement <u>cannot</u> be used to prove that $\triangle ABC$ is similar to $\triangle DEF$?

A) $\angle A \cong \angle D$ and $\angle B \cong \angle E$

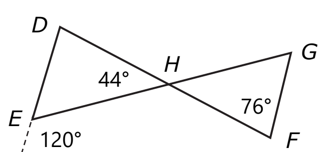
B) $\dfrac{AB}{DE} = \dfrac{BC}{EF} = \dfrac{AC}{DF}$

C) $\dfrac{AB}{DE} = \dfrac{AC}{DF}$ and $\angle A \cong \angle D$

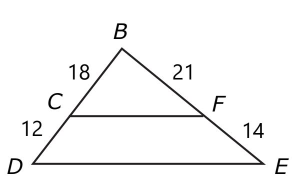
D) $\dfrac{AB}{DE} = \dfrac{BC}{EF}$ and $\angle C \cong \angle F$

For problems 3 through 5, determine if the triangles are similar. If so, state the reason and the similarity statement.

3.



4.



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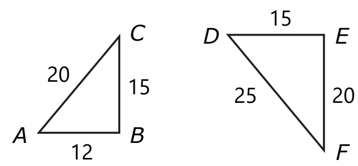
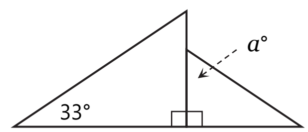
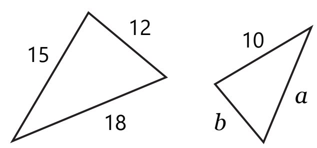


For problems 6 through 8, find the value of the variables that make the triangles similar.

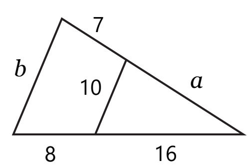
6.



7.

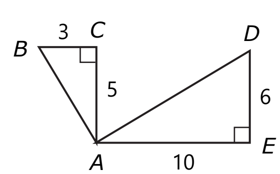


8.

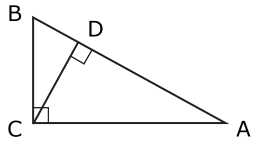


9. Complete the sentence.

A clockwise rotation of ? ° about A followed by a dilation with a scale factor of ? centered at A will map $\triangle ABC$ onto $\triangle DEF$. So, $\triangle ABC \sim \triangle DEF$.

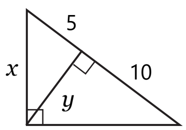


10. $\overline{CD}$ is an altitude of right $\triangle ABC$. Complete the similarity statement



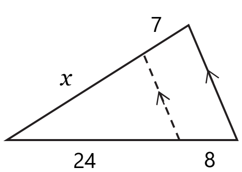
$\triangle ABC \sim \triangle$? $\sim \triangle$?

11. Find the values of x and y in simplest radical form.

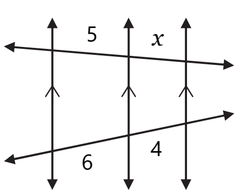


12. The altitude to the hypotenuse of a right triangle divides the hypotenuse into two segments. One segment is four times as long as the other. If the altitude is 10 cm long, what is the length of the shorter segment in simplest radical form?

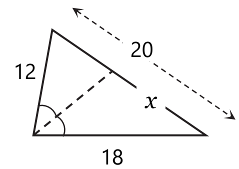
13. The dashed segment is parallel to a side of the triangle. Find the value of x.



14. Three parallel lines are cut by two transversals. Find the value of x.



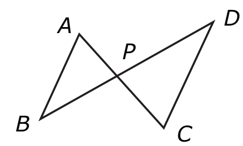
15. The dashed segment is an angle bisector of the triangle. Find the value of x.



16. Complete the proof.

Given: $\angle A \cong \angle C$

Prove: $\dfrac{PA}{PC} = \dfrac{PB}{PD}$

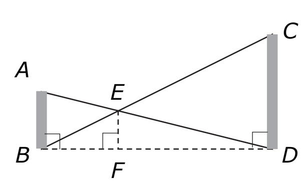


STATEMENTS	REASONS
1. $\angle A \cong \angle C$	1. Given
2. $\angle APB \cong \angle CPD$	2.
3. $\triangle APB \sim \triangle CPD$	3.
4. $\dfrac{PA}{PC} = \dfrac{PB}{PD}$	4.

17. A 20-meter flagpole casts a 24-meter shadow. A nearby building is 135 meters tall. How long is its shadow?

18. Kim places a mirror on the ground 20 ft from a tree and stands 5 ft from the mirror where she can see the reflection of the top of the tree. Kim is 4 ft tall. How tall is the tree?

19. (HONORS) Two pillars with heights 12 m and 24 m are connected with wires. The pillars are 48 m apart. How high is the intersection above the ground?



Hint: Let $EF = x$ and $BF = y$. Then you can set up two equations.