

Contents

LESSONS 1 ~ 180	13
QUIZZES 1 ~ 175	381
APPENDIX A Grading Sheets	425
APPENDIX B Bubble Answer Sheets	431

Completion Chart

ALGEBRA BASICS		Lesson	Quiz
LESSON 1	Order of Operations	<input type="checkbox"/>	<input type="checkbox"/>
LESSON 2	Algebraic Expressions	<input type="checkbox"/>	<input type="checkbox"/>
SOLVING LINEAR EQUATIONS		Lesson	Quiz
LESSON 3	Solving Basic Linear Equations	<input type="checkbox"/>	<input type="checkbox"/>
LESSON 4	Solving Multi-Step Linear Equations	<input type="checkbox"/>	<input type="checkbox"/>
LESSON 5	Solving Linear Equations with Fractions & Decimals	<input type="checkbox"/>	<input type="checkbox"/>
LESSON 6	Solving Absolute Value Equations	<input type="checkbox"/>	<input type="checkbox"/>
LESSON 7	Solving Linear Equations of All Types	<input type="checkbox"/>	<input type="checkbox"/>
LESSON 8	Number of Solutions to Linear Equations	<input type="checkbox"/>	<input type="checkbox"/>
LESSON 9	Applications of Linear Equations	<input type="checkbox"/>	<input type="checkbox"/>
LESSON 10	Applications of Linear Equations (Distance)	<input type="checkbox"/>	<input type="checkbox"/>
LESSON 11	Applications of Linear Equations (Mixture)	<input type="checkbox"/>	<input type="checkbox"/>
LESSON 12	Solving for a Variable	<input type="checkbox"/>	<input type="checkbox"/>
LESSON 13	Catch up and Review!	<input type="checkbox"/>	<input type="checkbox"/>
GRAPHING LINES		Lesson	Quiz
LESSON 14	Graphing Lines by Plotting Points	<input type="checkbox"/>	<input type="checkbox"/>
LESSON 15	Slope and Slope Formula	<input type="checkbox"/>	<input type="checkbox"/>
LESSON 16	Graphing Lines in Slope-Intercept Form	<input type="checkbox"/>	<input type="checkbox"/>
LESSON 17	Graphing Lines in Standard Form	<input type="checkbox"/>	<input type="checkbox"/>
LESSON 18	Graphing Lines Using Intercepts	<input type="checkbox"/>	<input type="checkbox"/>
LESSON 19	Horizontal, Vertical, Parallel, & Perpendicular Lines	<input type="checkbox"/>	<input type="checkbox"/>
LESSON 20	Graphing Absolute Value Equations	<input type="checkbox"/>	<input type="checkbox"/>
LESSON 21	Catch up and Review!	<input type="checkbox"/>	<input type="checkbox"/>

FINDING EQUATIONS OF LINES		Lesson	Quiz
LESSON 22	Finding Equations of Lines in Slope-Intercept Form	<input type="checkbox"/>	<input type="checkbox"/>
LESSON 23	Finding Equations of Lines in Slope-Intercept Form	<input type="checkbox"/>	<input type="checkbox"/>
LESSON 24	Finding Equations of Lines in Point-Slope Form	<input type="checkbox"/>	<input type="checkbox"/>
LESSON 25	Finding Equations of Lines in All Forms	<input type="checkbox"/>	<input type="checkbox"/>
LESSON 26	Applications of Slope-Intercept Form	<input type="checkbox"/>	<input type="checkbox"/>
LESSON 27	Applications of Standard Form	<input type="checkbox"/>	<input type="checkbox"/>
LESSON 28	Catch up and Review!	<input type="checkbox"/>	<input type="checkbox"/>
SYSTEMS OF LINEAR EQUATIONS		Lesson	Quiz
LESSON 29	Solving Linear Systems by Graphing	<input type="checkbox"/>	<input type="checkbox"/>
LESSON 30	Solving Linear Systems by Substitution	<input type="checkbox"/>	<input type="checkbox"/>
LESSON 31	Solving Linear Systems by Elimination	<input type="checkbox"/>	<input type="checkbox"/>
LESSON 32	Solving Linear Systems Using Any Method	<input type="checkbox"/>	<input type="checkbox"/>
LESSON 33	Number of Solutions to Linear Systems	<input type="checkbox"/>	<input type="checkbox"/>
LESSON 34	Applications of Linear Systems	<input type="checkbox"/>	<input type="checkbox"/>
LESSON 35	Applications of Linear Systems (Distance)	<input type="checkbox"/>	<input type="checkbox"/>
LESSON 36	Applications of Linear Systems (Mixture)	<input type="checkbox"/>	<input type="checkbox"/>
LESSON 37	Catch up and Review!	<input type="checkbox"/>	<input type="checkbox"/>
LINEAR INEQUALITIES		Lesson	Quiz
LESSON 38	Linear Inequalities in One Variable	<input type="checkbox"/>	<input type="checkbox"/>
LESSON 39	Compound Inequalities	<input type="checkbox"/>	<input type="checkbox"/>
LESSON 40	Absolute Value Inequalities	<input type="checkbox"/>	<input type="checkbox"/>
LESSON 41	Linear Inequalities in Two Variables	<input type="checkbox"/>	<input type="checkbox"/>
LESSON 42	Systems of Linear Inequalities	<input type="checkbox"/>	<input type="checkbox"/>
LESSON 43	Catch up and Review!	<input type="checkbox"/>	<input type="checkbox"/>
REVIEW & PSAT PRACTICE		Lesson	Quiz
LESSON 44	Review: 1 st Quarter	<input type="checkbox"/>	<input checked="" type="checkbox"/>
LESSON 45	Review: 1 st Quarter	<input type="checkbox"/>	<input checked="" type="checkbox"/>
LESSON 46	PSAT Practice	<input type="checkbox"/>	<input checked="" type="checkbox"/>
LESSON 47	PSAT Practice	<input type="checkbox"/>	<input checked="" type="checkbox"/>

FUNCTIONS		Lesson	Quiz
LESSON 48	Relations and Functions	<input type="checkbox"/>	<input type="checkbox"/>
LESSON 49	Domain and Range	<input type="checkbox"/>	<input type="checkbox"/>
LESSON 50	Function Notation	<input type="checkbox"/>	<input type="checkbox"/>
LESSON 51	Linear Functions	<input type="checkbox"/>	<input type="checkbox"/>
LESSON 52	Linear, Exponential, and Quadratic Functions	<input type="checkbox"/>	<input type="checkbox"/>
LESSON 53	Average Rate of Change	<input type="checkbox"/>	<input type="checkbox"/>
LESSON 54	Inverse of Linear Functions	<input type="checkbox"/>	<input type="checkbox"/>
LESSON 55	Catch up and Review!	<input type="checkbox"/>	<input type="checkbox"/>
VARIATION AND SEQUENCES		Lesson	Quiz
LESSON 56	Direct Variation	<input type="checkbox"/>	<input type="checkbox"/>
LESSON 57	Inverse Variation	<input type="checkbox"/>	<input type="checkbox"/>
LESSON 58	Arithmetic Sequences	<input type="checkbox"/>	<input type="checkbox"/>
LESSON 59	Geometric Sequences	<input type="checkbox"/>	<input type="checkbox"/>
LESSON 60	Recursive Formulas for Sequences	<input type="checkbox"/>	<input type="checkbox"/>
LESSON 61	Applications of Sequences	<input type="checkbox"/>	<input type="checkbox"/>
LESSON 62	Catch up and Review!	<input type="checkbox"/>	<input type="checkbox"/>
RADICALS		Lesson	Quiz
LESSON 63	Radicals	<input type="checkbox"/>	<input type="checkbox"/>
LESSON 64	Simplifying Radicals	<input type="checkbox"/>	<input type="checkbox"/>
LESSON 65	Simplifying Radicals with Variables	<input type="checkbox"/>	<input type="checkbox"/>
LESSON 66	Adding and Subtracting Radicals	<input type="checkbox"/>	<input type="checkbox"/>
LESSON 67	Multiplying and Dividing Radicals	<input type="checkbox"/>	<input type="checkbox"/>
LESSON 68	Solving Radical Equations	<input type="checkbox"/>	<input type="checkbox"/>
LESSON 69	Applications of Radicals and Radical Equations	<input type="checkbox"/>	<input type="checkbox"/>
LESSON 70	Catch up and Review!	<input type="checkbox"/>	<input type="checkbox"/>
EXPONENTS		Lesson	Quiz
LESSON 71	Zero and Negative Exponents	<input type="checkbox"/>	<input type="checkbox"/>
LESSON 72	Product and Quotient Rules of Exponents	<input type="checkbox"/>	<input type="checkbox"/>
LESSON 73	Power Rules of Exponents	<input type="checkbox"/>	<input type="checkbox"/>
LESSON 74	Simplifying Integer Exponents	<input type="checkbox"/>	<input type="checkbox"/>

LESSON 75	Scientific Notation	<input type="checkbox"/>	<input type="checkbox"/>
LESSON 76	Rational Exponents	<input type="checkbox"/>	<input type="checkbox"/>
LESSON 77	Simplifying Rational Exponents	<input type="checkbox"/>	<input type="checkbox"/>
LESSON 78	Exponential Growth and Decay	<input type="checkbox"/>	<input type="checkbox"/>
LESSON 79	Catch up and Review!	<input type="checkbox"/>	<input type="checkbox"/>

POLYNOMIALS

Lesson Quiz

LESSON 80	Adding and Subtracting Polynomials	<input type="checkbox"/>	<input type="checkbox"/>
LESSON 81	Multiplying Monomials and Binomials	<input type="checkbox"/>	<input type="checkbox"/>
LESSON 82	Multiplying Polynomials	<input type="checkbox"/>	<input type="checkbox"/>
LESSON 83	Special Products of Binomials	<input type="checkbox"/>	<input type="checkbox"/>
LESSON 84	Multiplying Binomials and Special Products	<input type="checkbox"/>	<input type="checkbox"/>
LESSON 85	Dividing Polynomials	<input type="checkbox"/>	<input type="checkbox"/>
LESSON 86	Dividing Polynomials	<input type="checkbox"/>	<input type="checkbox"/>
LESSON 87	Catch up and Review!	<input type="checkbox"/>	<input type="checkbox"/>

REVIEW & PSAT PRACTICE

Lesson Quiz

LESSON 88	Review: 1 st Quarter	<input type="checkbox"/>	<input checked="" type="checkbox"/>
LESSON 89	Review: 2 nd Quarter	<input type="checkbox"/>	<input checked="" type="checkbox"/>
LESSON 90	Review: 2 nd Quarter	<input type="checkbox"/>	<input checked="" type="checkbox"/>
LESSON 91	PSAT Practice	<input type="checkbox"/>	<input checked="" type="checkbox"/>
LESSON 92	PSAT Practice	<input type="checkbox"/>	<input checked="" type="checkbox"/>

FACTORING POLYNOMIALS

Lesson Quiz

LESSON 93	Factoring Polynomials Using GCFs	<input type="checkbox"/>	<input type="checkbox"/>
LESSON 94	Factoring Polynomials by Grouping	<input type="checkbox"/>	<input type="checkbox"/>
LESSON 95	Factoring Simple Quadratics	<input type="checkbox"/>	<input type="checkbox"/>
LESSON 96	Factoring Simple Quadratics	<input type="checkbox"/>	<input type="checkbox"/>
LESSON 97	Factoring Quadratics	<input type="checkbox"/>	<input type="checkbox"/>
LESSON 98	Factoring Quadratics	<input type="checkbox"/>	<input type="checkbox"/>
LESSON 99	Factoring Special Products	<input type="checkbox"/>	<input type="checkbox"/>
LESSON 100	Factoring Quadratics in Any Form	<input type="checkbox"/>	<input type="checkbox"/>
LESSON 101	Factoring Polynomials Completely	<input type="checkbox"/>	<input type="checkbox"/>
LESSON 102	Catch up and Review!	<input type="checkbox"/>	<input type="checkbox"/>

SOLVING QUADRATIC EQUATIONS		Lesson	Quiz
LESSON 103	Solving Quadratics by Taking Square Roots	<input type="checkbox"/>	<input type="checkbox"/>
LESSON 104	Solving Quadratics by Taking Square Roots	<input type="checkbox"/>	<input type="checkbox"/>
LESSON 105	Solving Quadratics by Factoring	<input type="checkbox"/>	<input type="checkbox"/>
LESSON 106	Solving Quadratics by Factoring	<input type="checkbox"/>	<input type="checkbox"/>
LESSON 107	Solving Quadratics by Completing the Square	<input type="checkbox"/>	<input type="checkbox"/>
LESSON 108	Solving Quadratics by Completing the Square	<input type="checkbox"/>	<input type="checkbox"/>
LESSON 109	Solving Quadratics by the Quadratic Formula	<input type="checkbox"/>	<input type="checkbox"/>
LESSON 110	Solving Quadratics by the Quadratic Formula	<input type="checkbox"/>	<input type="checkbox"/>
LESSON 111	Solving Quadratics Using Any Method	<input type="checkbox"/>	<input type="checkbox"/>
LESSON 112	Number of Solutions to Quadratic Equations	<input type="checkbox"/>	<input type="checkbox"/>
LESSON 113	Applications of Quadratic Equations	<input type="checkbox"/>	<input type="checkbox"/>
LESSON 114	Catch up and Review!	<input type="checkbox"/>	<input type="checkbox"/>
GRAPHING PARABOLAS		Lesson	Quiz
LESSON 115	Graphing Parabolas by Plotting Points	<input type="checkbox"/>	<input type="checkbox"/>
LESSON 116	Graphing Parabolas Using the Vertex Formula	<input type="checkbox"/>	<input type="checkbox"/>
LESSON 117	Graphing Parabolas in Vertex Form	<input type="checkbox"/>	<input type="checkbox"/>
LESSON 118	Graphing Parabolas in Intercept Form	<input type="checkbox"/>	<input type="checkbox"/>
LESSON 119	Graphing Parabolas in Standard Form	<input type="checkbox"/>	<input type="checkbox"/>
LESSON 120	Graphing Parabolas in All Forms	<input type="checkbox"/>	<input type="checkbox"/>
LESSON 121	Finding Equations of Parabolas	<input type="checkbox"/>	<input type="checkbox"/>
LESSON 122	Transforming Parabolas	<input type="checkbox"/>	<input type="checkbox"/>
LESSON 123	Applications of Quadratic Functions	<input type="checkbox"/>	<input type="checkbox"/>
LESSON 124	Catch up and Review!	<input type="checkbox"/>	<input type="checkbox"/>
RATIONAL EXPRESSIONS AND EQUATIONS		Lesson	Quiz
LESSON 125	Simplifying Rational Expressions	<input type="checkbox"/>	<input type="checkbox"/>
LESSON 126	Multiplying and Dividing Rational Expressions	<input type="checkbox"/>	<input type="checkbox"/>
LESSON 127	Adding and Subtracting Rational Expressions	<input type="checkbox"/>	<input type="checkbox"/>
LESSON 128	Solving Rational Equations	<input type="checkbox"/>	<input type="checkbox"/>
LESSON 129	Solving Rational Equations	<input type="checkbox"/>	<input type="checkbox"/>
LESSON 130	Applications of Rational Equations	<input type="checkbox"/>	<input type="checkbox"/>
LESSON 131	Catch up and Review!	<input type="checkbox"/>	<input type="checkbox"/>

REVIEW & PSAT PRACTICE		Lesson	Quiz
LESSON 132	Review: 1 st Quarter	<input type="checkbox"/>	<input checked="" type="checkbox"/>
LESSON 133	Review: 2 nd Quarter	<input type="checkbox"/>	<input checked="" type="checkbox"/>
LESSON 134	Review: 3 rd Quarter	<input type="checkbox"/>	<input checked="" type="checkbox"/>
LESSON 135	Review: 3 rd Quarter	<input type="checkbox"/>	<input checked="" type="checkbox"/>
LESSON 136	PSAT Practice	<input type="checkbox"/>	<input checked="" type="checkbox"/>
LESSON 137	PSAT Practice	<input type="checkbox"/>	<input checked="" type="checkbox"/>

DATA ANALYSIS		Lesson	Quiz
LESSON 138	Measures of Center	<input type="checkbox"/>	<input type="checkbox"/>
LESSON 139	Measures of Variation	<input type="checkbox"/>	<input type="checkbox"/>
LESSON 140	Box-and-Whisker Plots	<input type="checkbox"/>	<input type="checkbox"/>
LESSON 141	Visualizing Data	<input type="checkbox"/>	<input type="checkbox"/>
LESSON 142	Shapes of Distributions	<input type="checkbox"/>	<input type="checkbox"/>
LESSON 143	Scatter Plots	<input type="checkbox"/>	<input type="checkbox"/>
LESSON 144	Two-Way Frequency Tables	<input type="checkbox"/>	<input type="checkbox"/>
LESSON 145	Catch up and Review!	<input type="checkbox"/>	<input type="checkbox"/>

PROBABILITY		Lesson	Quiz
LESSON 146	Theoretical and Experimental Probability	<input type="checkbox"/>	<input type="checkbox"/>
LESSON 147	Sample Spaces	<input type="checkbox"/>	<input type="checkbox"/>
LESSON 148	Fundamental Counting Principle	<input type="checkbox"/>	<input type="checkbox"/>
LESSON 149	Probability of Independent Events	<input type="checkbox"/>	<input type="checkbox"/>
LESSON 150	Probability of Dependent Events	<input type="checkbox"/>	<input type="checkbox"/>
LESSON 151	Probability of Disjoint and Overlapping Events	<input type="checkbox"/>	<input type="checkbox"/>
LESSON 152	Permutations and Combinations	<input type="checkbox"/>	<input type="checkbox"/>
LESSON 153	Permutations and Combinations	<input type="checkbox"/>	<input type="checkbox"/>
LESSON 154	Permutations, Combinations, and Probability	<input type="checkbox"/>	<input type="checkbox"/>
LESSON 155	Catch up and Review!	<input type="checkbox"/>	<input type="checkbox"/>

TEST TAKING TIPS & SELF-REVIEW		Lesson	Quiz
LESSON 156	Test Preparation Tips & Self-Review	<input type="checkbox"/>	<input checked="" type="checkbox"/>
LESSON 157	Test Taking Tips & Self-Review	<input type="checkbox"/>	<input checked="" type="checkbox"/>

REVIEW: ALL TOPICS IN ALGEBRA 1		Lesson	Quiz
LESSON 158	Review: Solving Linear Equations & Inequalities	<input type="checkbox"/>	<input type="checkbox"/>
LESSON 159	Review: Graphing Linear Functions & Inequalities	<input type="checkbox"/>	<input type="checkbox"/>
LESSON 160	Review: Systems of Linear Equations & Inequalities	<input type="checkbox"/>	<input type="checkbox"/>
LESSON 161	Review: Functions, Variation, and Sequences	<input type="checkbox"/>	<input type="checkbox"/>
LESSON 162	Review: Radicals	<input type="checkbox"/>	<input type="checkbox"/>
LESSON 163	Review: Exponents	<input type="checkbox"/>	<input type="checkbox"/>
LESSON 164	Review: Polynomials and Factoring	<input type="checkbox"/>	<input type="checkbox"/>
LESSON 165	Review: Solving Quadratic Equations	<input type="checkbox"/>	<input type="checkbox"/>
LESSON 166	Review: Graphing Quadratic Functions	<input type="checkbox"/>	<input type="checkbox"/>
LESSON 167	Review: Rational Expressions and Equations	<input type="checkbox"/>	<input type="checkbox"/>
LESSON 168	Review: Data Analysis	<input type="checkbox"/>	<input type="checkbox"/>
LESSON 169	Review: Probability	<input type="checkbox"/>	<input type="checkbox"/>
REVIEW: SOLVING ALL TYPES OF EQUATIONS		Lesson	Quiz
LESSON 170	Review: Solving Linear Equations	<input type="checkbox"/>	<input type="checkbox"/>
LESSON 171	Review: Solving Linear Inequalities	<input type="checkbox"/>	<input type="checkbox"/>
LESSON 172	Review: Solving Systems of Linear Equations	<input type="checkbox"/>	<input type="checkbox"/>
LESSON 173	Review: Solving Radical Equations	<input type="checkbox"/>	<input type="checkbox"/>
LESSON 174	Review: Solving Quadratic Equations	<input type="checkbox"/>	<input type="checkbox"/>
LESSON 175	Review: Solving Rational Equations	<input type="checkbox"/>	<input type="checkbox"/>
FINAL EXAM WEEK		Lesson	Quiz
LESSON 176	Study for Final Exam	<input type="checkbox"/>	<input checked="" type="checkbox"/>
LESSON 177	Study for Final Exam	<input type="checkbox"/>	<input checked="" type="checkbox"/>
LESSON 178	Study for Final Exam	<input type="checkbox"/>	<input checked="" type="checkbox"/>
LESSON 179	Study for Final Exam	<input type="checkbox"/>	<input checked="" type="checkbox"/>
LESSON 180	Final Exam	<input type="checkbox"/>	<input checked="" type="checkbox"/>

LESSON 4 Solving Multi-Step Linear Equations

REFRESH YOUR SKILLS

Simplify by using the distributive property and combining like terms. Review Lesson 2 if needed.

1. $5x - 6 + 2x + 9 - x$

2. $9 + 5(3x - 2) - 7x$

LEARN A SKILL: SOLVING LINEAR EQUATIONS WITH VARIABLES ON BOTH SIDES

When solving equations with variables on both sides, first use inverse operations to collect all variables on one side and all constants on the other. Then solve as usual.

→ **EXAMPLE** Solve $2x + 3 = 9 - 4x$.

$$\begin{array}{r} 2x + 3 = 9 - 4x \\ + 4x \quad + 4x \quad \text{Add } 4x \text{ to both sides.} \\ \hline 6x + 3 = 9 \\ - 3 \quad - 3 \quad \text{Subtract 3 from both sides.} \\ \hline 6x = 6 \\ \div 6 \quad \div 6 \quad \text{Divide both sides by 6.} \\ \hline x = 1 \quad \text{Find the solution.} \end{array}$$

Check:

$$2 \cdot 1 + 3 = 9 - 4 \cdot 1 \quad \text{Check the solution.}$$

→ **TRY IT** Solve.

3. $4x - 8 = 2x$

4. $-2x = 6 + x$

5. $3x + 5 = 4x - 2$

6. $7x - 9 = -9x + 7$

LEARN A SKILL: SOLVING MULTI-STEP LINEAR EQUATIONS

The first step in solving any linear equation is to simplify each side of the equation by removing parentheses and combining like terms. Then you can solve the equation as usual.

→ **EXAMPLE** Solve $1 + 2x - 5 = 2(1 - x)$.

$$\begin{array}{r} 1 + 2x - 5 = 2(1 - x) \\ \quad \quad \quad 2 - 2x \\ \hline 2x - 4 = 2 - 2x \quad \text{Simplify each side.} \\ + 2x \quad + 2x \quad \text{Add } 2x \text{ to both sides.} \\ \hline 4x - 4 = 2 \\ + 4 \quad + 4 \quad \text{Add 4 to both sides.} \\ \hline 4x = 6 \\ \div 4 \quad \div 4 \quad \text{Divide both sides by 4.} \\ \hline x = \frac{3}{2} \quad \text{We usually leave a} \\ \quad \quad \quad \text{fraction improper.} \end{array}$$

Check:

$$1 + 2 \cdot \frac{3}{2} - 5 = 2\left(1 - \frac{3}{2}\right) \quad \text{Check the solution.}$$

→ **TRY IT** Solve.

7. $3(1 - x) = 9$

8. $5 + 3(2x - 3) = 2$

9. $-4(x + 1) + 9 = -7$

10. $6x + 5 = 5(x - 3)$

11. $7(x - 4) + 2 = 6 - x$

12. $3x + x = 4(x + 3) + 4x$

□ **EXERCISE YOUR SKILLS**

Solve. Reduce fractions but leave them improper. Check your solutions.

13. $4x + 8 = 2x$

14. $-4x = 40 + x$

15. $8x - 3x + 5 = 0$

16. $5x + 2x - 9 = 6$

17. $x + 8 - 4x = -16$

18. $9x - 12 = -x + 8$

19. $20 - 3x = 5x - 2$

20. $3x - 9 = -5x + 7$

21. $4x - 8 - x = -29$

22. $4x + 5 = 10x - 13$

23. $9(3 + x) + 5 = 11$

24. $3(6 - 3x) = -36$

25. $3 + 7(2x - 8) = 17$

26. $2(4 + x) - 4(x - 3) = 7$

27. $4(x - 3) = 3x + 5$

28. $5x = 2(1 - 4x) + 6x$

29. $7x + 4 = 2(x - 4) - 3$

30. $3(2x + 3) = -2x - 15$

31. $4x + x - 5 = 2(4x + 1)$

32. $5x + 4(2 - 3x) = 2x + 6$

33. $2 + x - 8 = 2x + 3(2 - x)$

34. $2(3x + 4) + 11 = 9(x + 2)$

□ **CHALLENGE YOURSELF**

Solve. Reduce fractions but leave them improper. Check your solutions.

35. $\frac{1}{3}x + 5 = \frac{2}{3}x - 2$

36. $\frac{1}{2}(x - 2) + \frac{3}{4} = \frac{1}{4}$

37. $\frac{3}{4} + 4x = \frac{1}{2}(4x + 1)$

38. $\frac{5}{2}x - \frac{1}{5} = \frac{1}{2}(x + 2) + \frac{1}{5}$

LESSON 9 Applications of Linear Equations

□ LEARN A SKILL: SOLVING LINEAR EQUATION WORD PROBLEMS

There are many types of word problems that require algebra: number problems, geometry problems, money problems, percentage problems, distance problems, mixture problems, and so on. You may find some more difficult than others, but they all can be solved using the same strategy. To solve a word problem using algebra, 1) define a variable, 2) set up an equation to model the given situation, 3) solve the equation as usual, and then 4) answer what's being asked.

→ EXAMPLE Consecutive integers

The sum of three consecutive even integers is 12. Find the integers.

1. Let x = the first even integer
2. $x + 2$ = the second even integer
 $x + 4$ = the third even integer
The sum is 12, so $x + (x + 2) + (x + 4) = 12$.
3. Solve for x , and you get $x = 2$.
4. The numbers are 2, 4, and 6.

→ EXAMPLE Percent

The price of an apple rose by 15% to \$1.38/lb. What was the original price?

1. Let x = the original price
2. $0.15x$ = the price increase
The new price = the original price + the price increase, so $1.38 = x + 0.15x$.
3. Solve for x , and you get $x = 1.2$.
4. The original price was \$1.20/lb.

→ EXAMPLE Age

Dale is 6 years older than Kate. Three years ago, Dale was twice as old as Kate. How old are they now?

1. Let x = Kate's age now
2. $x + 6$ = Dale's age now
 $x - 3$ = Kate's age 3 years ago
 $(x + 6) - 3$ = Dale's age 3 years ago
Dale's age 3 years ago = twice Kate's age 3 years ago, so $(x + 6) - 3 = 2(x - 3)$.
3. Solve for x , and you get $x = 9$.
4. Kate is 9 years old. Dale is 15 years old.

→ TRY IT Solve.

1. The sum of two consecutive even integers is 26. Find the two integers.
2. The sum of two consecutive odd integers is 32. Find the two integers.
3. The sum of three consecutive integers is 27. Find the integers.

→ TRY IT Solve.

4. Cammy bought a jacket at \$43.50. The price was 25% off the regular price. What was the regular price?
5. Jim bought a pair of pants at \$16.80. The price was 40% off the regular price. What was the regular price?

→ TRY IT Solve.

6. Jamie is 5 years older than Nicole. Two years ago, Jamie was twice as old as Nicole. How old are they now?
7. Mia's father is 42 years old. Six years ago, he was six times as old as Mia. How old is Mia?
8. Ellen is 10 years younger than Max. In two years, Max will be twice as old as Ellen. How old are they now?

→ **EXAMPLE** Geometry

The length of a rectangle is twice its width. The perimeter is 18 feet. Find the dimensions of the rectangle.

1. Let x = the width of the rectangle
2. $2x$ = the length of the rectangle
Perimeter = $2(\text{length} + \text{width})$,
so $2(x + 2x) = 18$.
3. Solve for x , and you get $x = 3$.
4. The rectangle is 3 feet by 6 feet.

→ **EXAMPLE** Coins

Olivia has \$0.90 in dimes and nickels. She has three more nickels than dimes. How many coins of each type does she have?

1. Let x = the number of dimes
2. $x + 3$ = the number of nickels
Total value = x dimes at \$0.10 each +
 $(x + 3)$ nickels at \$0.05 each,
so $0.10x + 0.05(x + 3) = 0.90$.
3. Solve for x , and you get $x = 5$.
4. Olivia has 5 dimes and 8 nickels.

→ **TRY IT** Solve.

9. The length of a rectangle is three times its width. The perimeter is 40 feet. Find the dimensions of the rectangle.
10. The length of a rectangle is 5 cm less than three times its width. The perimeter is 22 cm. Find the dimensions of the rectangle.

→ **TRY IT** Solve.

11. Emma has \$0.95 in dimes and nickels. She has five more dimes than nickels. How many coins of each type does she have?
12. Joey has \$1.60 in quarters and nickels. He has two more nickels than quarters. How many coins of each type does he have?

□ **EXERCISE YOUR SKILLS**

For each problem, 1) define a variable, 2) set up an equation, 3) solve the equation, and 4) answer what's being asked. Show your work in your notebook.

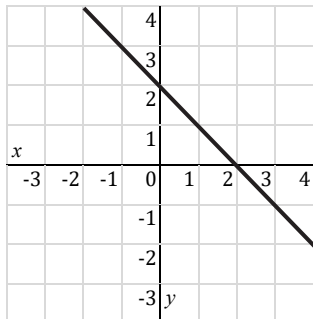
13. One integer is 5 less than twice another. Their sum is 25. Find the two integers.
14. The sum of three consecutive odd integers is 27. Find the integers.
15. A company's stock price dropped by 10% to \$108 per share. What was the previous price per share?
16. Currently, Joey is three times as old as Anna. In five years, Joey will be twice as old as Anna. How old is Joey? How old is Anna?
17. Two sides of a triangle are equal in length and twice the length of the shortest side. The perimeter is 45 inches. Find the dimensions of the triangle.
18. Max has one-, five-, and ten-dollar bills totaling \$82. He has twice as many fives as ones and three times as many tens as ones. How many bills of each type does he have?

LESSON 28 Catch up and Review!

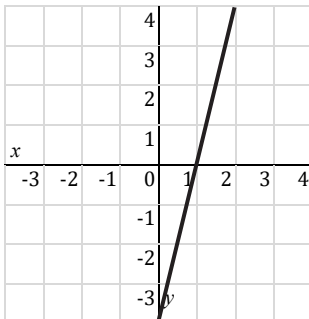
Catch up if you are behind. Use the review problems below to make sure you're on track.

LESSON 22 Find an equation of each line in slope-intercept form.

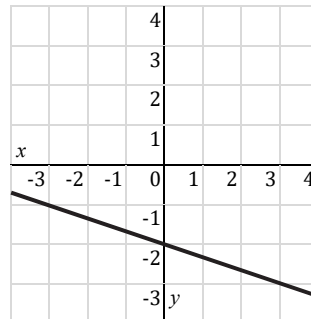
1.



2.



3.



LESSON 23 Find an equation of each line in slope-intercept form.

4. slope = 2; through (0, 4)

5. slope = 5; through (2, 9)

6. through (4, 9) and (-1, -6)

7. through (3, 2) and (4, 0)

LESSON 24 Find an equation of each line in point-slope form. Use the first point for point-slope form when given two points.

8. slope = -1; through (3, 0)

9. slope = 2; through (5, 4)

10. through (-2, -7) and (1, 5)

11. through (3, -8) and (2, -3)

LESSON 25 Find an equation of each line in standard form. Use only integers and the smallest possible positive integer coefficient for x .

12. slope = 5; through (2, 4)

13. slope = $\frac{4}{5}$; through (-5, 3)

14. through (5, 3) and (-5, 1)

15. through (-4, 8) and (6, 3)

LESSON 26 Solve.

16. A taxi charges a flat fee of \$5 and \$1.60 per mile.
- a. Write an equation representing the total cost, y , of riding the taxi for x miles.
 - b. How much will a taxi ride cost for 15 miles?
 - c. If a taxi ride cost \$45, how many miles did the taxi travel?
17. An internet service provider charges \$32 per month plus an initial set-up fee of \$58.
- a. Write an equation representing the total cost, y , after x months of service.
 - b. How much will it cost after 5 months of service?
 - c. If a customer spent a total of \$442, how long was the service provided?

LESSON 27 Solve.

18. Mia has x quarters and y dimes amounting to \$2.30.
- a. Write an equation relating x and y .
 - b. If she has 6 quarters, how many dimes does she have?
19. At a grocery, Rodney bought 3 bags of onions and 2 bags of potatoes. He spent \$20 in total. Onions cost x per bag, and potatoes cost y per bag.
- a. Write an equation relating x and y .
 - b. If each bag of potatoes costs \$5.50, how much does each bag of onions cost?

REFRESH YOUR SKILLS: PRE-ALGEBRA

Brush up on the topics covered in Pre-Algebra.

20. What is 20% of 80?
21. Convert 40% to a fraction. Simplify your answer.
22. You cut 17 feet from a 50-foot wire. What is the percent decrease in length?
23. One meter is approximately 3 feet. How many centimeters are there in 1.2 feet?

LESSON 46 PSAT Practice

This is a timed practice test. Get a timer, a bubble answer sheet (provided in Appendix B), and blank sheets of paper for your calculations. When you are ready, set the timer for **25 minutes** and begin. Do not use a calculator. Mark all your answers on the answer sheet. Only answers marked on the answer sheet can be scored. After the test, make sure you review what you missed.

1. Which of the following is equivalent to the expression $4 - 3(x - 2) + 2x$?

A) $-x - 2$ B) $-x + 10$
C) $5x - 2$ D) $5x + 10$

2. $2x - 5 = 4 - x$

If x is the solution to the equation above, what is the value of $x + 3$?

A) -3 B) 0 C) 3 D) 6

3. $2x + y = 2$ and $x + 3y = -9$

Which ordered pair (x, y) satisfies the system of equations above?

A) $(-2, 6)$ B) $(0, -3)$
C) $(1, 0)$ D) $(3, -4)$

4. Which of the following is equivalent to the inequality $14 - 2x > 3(x - 2)$?

A) $x > -4$ B) $x > 4$
C) $x < 4$ D) $x > 8$

5. $7 + 3x = 3(x + c) - 5$

Which value of c makes the equation above have infinitely many solutions?

A) 0 B) 1 C) 2 D) 4

6. $|3 - 2x| = 7$

If a and b are the solutions to the equation above, what is the value of $a + b$?

A) 3 B) 5 C) 7 D) 9

7. $x - y = 5$ and $x + 2y = -1$

If (p, q) is a solution to the system above, what is the value of p ?

A) -1 B) -2 C) 3 D) 6

8. $F = \frac{9}{5}C + 32$

The formula above gives the Fahrenheit temperature F for a given Celsius temperature C . Which formula gives the Celsius temperature C for a given Fahrenheit temperature F ?

A) $C = \frac{5}{9}F - 32$ B) $C = \frac{5}{9}(F - 32)$

C) $C = \frac{9}{5}F - 32$ D) $C = \frac{9}{5}(F - 32)$

9. A recipe calls for 2 quarts of milk, but Josh has only 2 cups of milk. How much more milk does he need in cups? (1 quart = 2 pints and 1 pint = 2 cups)

A) 2 B) 4 C) 6 D) 8

Continue to the next page.

10. A hiking club has 45 members. The ratio of males to females is 2:3. How many males are in the club?

A) 9 B) 18 C) 27 D) 30

11. The line with the equation $x - y = 3$ does NOT pass through which of the four quadrants?

A) I B) II C) III D) IV

12. $y = 50 + 30x$

The equation above models the total cost, y , that an electrician charges for x hours of service. The total cost consists of a one-time fee plus an hourly charge. If the equation is graphed in the xy -plane, what is indicated by the y -intercept of the graph?

A) A one-time fee of \$30
 B) A one-time fee of \$50
 C) An hourly charge of \$30
 D) An hourly charge of \$50

13. An airplane 8 kilometers above the ground begins descending at an average speed of 350 meters per minute. Which expression represents the altitude of the plane, in kilometers, after t minutes?

A) $8 - 0.35t$ B) $8 - 350t$
 C) $8000 - 0.35t$ D) $8000 - 350t$

14. Mark is 18 years old now. Two years ago, Mark was twice as old as Kate. How many years older than Kate is Mark?

A) 6 B) 8 C) 10 D) 12

15. Natalie bought a hat using a \$2 coupon off the regular price. With sales tax of 5% added, she paid \$8.40 in total. Which equation can be used to determine the regular price, x , of the hat?

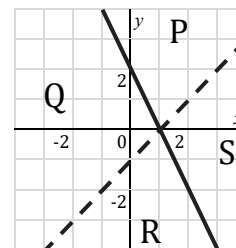
A) $1.05x + 2 = 8.4$
 B) $1.05x - 2 = 8.4$
 C) $1.05(x + 2) = 8.4$
 D) $1.05(x - 2) = 8.4$

16. A group of x adults and y children went to see a movie. Movie tickets cost \$8 for adults and \$6 for children. The group bought 10 tickets and paid \$72 in total. Which system of equations represents the relationship between x and y ?

A) $x + y = 10$ and $8x + 6y = 72$
 B) $x + y = 10$ and $8x + 6y = 10 \cdot 72$
 C) $x + y = 72$ and $8x + 6y = 10$
 D) $x + y = 72$ and $8x + 6y = 10 \cdot 72$

17. $x - y > 1$ and $2x + y \leq 2$

The system of inequalities above is graphed below. Which region represents the solution to the system?



A) Region P
 B) Region Q
 C) Region R
 D) Region S

STOP

This is the end of the test. If you finish before time is up, check your work.

LESSON 64 Simplifying Radicals

REFRESH YOUR SKILLS

Evaluate. Review Lesson 63 if needed.

1. $\sqrt{25}$

2. $\sqrt{81}$

3. $\sqrt[3]{-27}$

4. $\sqrt[5]{32}$

LEARN A SKILL: SIMPLIFYING SQUARE ROOTS

Simplifying a radical is to make the radicand, the number under the radical sign, as small as possible. There are two rules that can be used when simplifying radicals. The **product rule for radicals** says that the product of two radicals is the radical of the product. The **quotient rule for radicals** says that the quotient of two radicals is the radical of the quotient.

$$\text{Product rule: } \sqrt[n]{ab} = \sqrt[n]{a} \cdot \sqrt[n]{b}$$

$$\text{Quotient rule: } \sqrt[n]{\frac{a}{b}} = \frac{\sqrt[n]{a}}{\sqrt[n]{b}}$$

To simplify a square root, look for the largest perfect square factor of the radicand and pull its square root out of the radical. For example, in the radical $\sqrt{20}$ below, 4 is the largest perfect square factor of 20 and so we can pull its square root 2 out of the radical.

→ **EXAMPLE** Simplify.

$$\sqrt{20} = \sqrt{4 \cdot 5} = \sqrt{4} \cdot \sqrt{5} = 2\sqrt{5}$$

$$2\sqrt{18} = 2 \cdot \sqrt{9} \cdot \sqrt{2} = 6\sqrt{2}$$

$$\sqrt{\frac{8}{49}} = \frac{\sqrt{8}}{\sqrt{49}} = \frac{\sqrt{4} \cdot \sqrt{2}}{7} = \frac{2\sqrt{2}}{7}$$

→ **TRY IT** Simplify.

5. $\sqrt{32}$

6. $4\sqrt{28}$

7. $\sqrt{\frac{27}{25}}$

8. $\sqrt{\frac{45}{64}}$

LEARN A SKILL: SIMPLIFYING CUBE ROOTS

To simplify a cube root, look for the largest perfect cube factor of the radicand and pull its cube root out of the radical.

→ **EXAMPLE** Simplify.

$$3\sqrt[3]{16} = 3 \cdot \sqrt[3]{8} \cdot \sqrt[3]{2} = 6\sqrt[3]{2}$$

$$\sqrt[3]{-54} = \sqrt[3]{-27} \cdot \sqrt[3]{2} = -3\sqrt[3]{2}$$

$$\sqrt[3]{\frac{40}{27}} = \frac{\sqrt[3]{40}}{\sqrt[3]{27}} = \frac{\sqrt[3]{8} \cdot \sqrt[3]{5}}{\sqrt[3]{27}} = \frac{2\sqrt[3]{5}}{3}$$

→ **TRY IT** Simplify.

9. $\sqrt[3]{24}$

10. $5\sqrt[3]{-40}$

11. $\sqrt[3]{\frac{81}{64}}$

12. $\sqrt[3]{\frac{32}{125}}$

□ **LEARN A SKILL: SIMPLIFYING RADICALS USING PRIME FACTORIZATION**

You can also use prime factorization to simplify radicals. To simplify a square root using prime factorization, factor the radicand into its prime factors and pull out any factors that occur in pairs. To simplify a cube root, pull out any factors that occur three times.

→ **EXAMPLE** Simplify using prime factorization.

$$\sqrt{48} = \sqrt{2^2 \cdot 2^2 \cdot 3} = \sqrt{2^2} \cdot \sqrt{2^2} \cdot \sqrt{3} = 2 \cdot 2 \cdot \sqrt{3} = 4\sqrt{3}$$

$$\sqrt[3]{432} = \sqrt[3]{2^3 \cdot 3^3 \cdot 2} = \sqrt[3]{2^3} \cdot \sqrt[3]{3^3} \cdot \sqrt[3]{2} = 2 \cdot 3 \cdot \sqrt[3]{2} = 6\sqrt[3]{2}$$

→ **TRY IT** Simplify.

13. $\sqrt{80}$

14. $\sqrt{112}$

15. $\sqrt[3]{135}$

16. $\sqrt{250}$

□ **EXERCISE YOUR SKILLS**

Simplify each square root.

17. $\sqrt{8}$

18. $\sqrt{12}$

19. $\sqrt{18}$

20. $\sqrt{27}$

21. $\sqrt{24}$

22. $\sqrt{45}$

23. $\sqrt{50}$

24. $\sqrt{63}$

25. $7\sqrt{20}$

26. $2\sqrt{28}$

27. $9\sqrt{48}$

28. $2\sqrt{72}$

29. $\sqrt{\frac{18}{25}}$

30. $\sqrt{\frac{45}{16}}$

31. $3\sqrt{\frac{50}{81}}$

32. $4\sqrt{\frac{27}{64}}$

Simplify each cube root.

33. $\sqrt[3]{40}$

34. $\sqrt[3]{54}$

35. $2\sqrt[3]{81}$

36. $5\sqrt[3]{-16}$

37. $\sqrt[3]{\frac{48}{125}}$

38. $\sqrt[3]{\frac{135}{64}}$

39. $6\sqrt[3]{\frac{32}{27}}$

40. $10\sqrt[3]{\frac{81}{1000}}$

LESSON 90 Review: 2nd Quarter

Let's review the topics covered in the second quarter.

LESSON 63 Evaluate.

1. $\sqrt{25}$

2. $\sqrt[3]{27}$

3. $\sqrt[4]{16}$

4. $\sqrt[5]{-100,000}$

LESSONS 64–65 Simplify. Assume that all variables are positive.

5. $\sqrt{24}$

6. $\sqrt[3]{54}$

7. $\sqrt{x^3}$

8. $\sqrt[3]{x^3y^4}$

9. $\sqrt{\frac{27}{16}}$

10. $\sqrt[3]{\frac{24}{125}}$

11. $\sqrt{\frac{8x}{y^2}}$

12. $\sqrt[3]{\frac{8x^3}{y^6}}$

LESSONS 66–67 Simplify. Rationalize all denominators.

13. $\sqrt{50} + \sqrt{72}$

14. $7\sqrt{48} - 3\sqrt{12}$

15. $2\sqrt{5} \cdot \sqrt{15}$

16. $\sqrt{10x^3} \cdot \sqrt{5x}$

17. $\frac{10}{\sqrt{5}} + \sqrt{80}$

18. $\frac{\sqrt{72x^2y}}{2\sqrt{9xy}}$

LESSON 68 Solve. Check for extraneous solutions.

19. $\sqrt{x} + 2 = 3$

20. $\sqrt{x-7} + 4 = 6$

21. $8 - \sqrt{2x+5} = 5$

22. $3\sqrt{x-1} = -\sqrt{2x+5}$

23. $9 + \sqrt[3]{x-15} = 6$

24. $\sqrt[3]{x} = \sqrt[3]{5x+2}$

LESSON 69 Find the missing side length of each right triangle.

25. Legs = 5 and $5\sqrt{3}$

26. Hypotenuse = $6\sqrt{2}$ and one leg = 6

LESSON 71 Evaluate.

27. 9^0

28. 5^{-2}

29. $4^5 \cdot 2^{-9}$

30. $3^6 \cdot 6^4 \cdot 9^{-5}$

LESSONS 72–74 Simplify using positive exponents.

31. $5x^{-4} \cdot 2x^3$

32. $x^{-2} \cdot (2x^3)^4$

33. $\frac{7x^{-4}x^2}{x^5x^3}$

34. $\frac{27x^2}{(3x^{-3})^2}$

LESSON 76 Evaluate.

35. $16^{1/2}$

36. $(-8)^{1/3}$

37. $81^{-1/4}$

38. $\left(\frac{1}{125}\right)^{-2/3}$

LESSON 77 Simplify using positive exponents.

39. $x^{1/3} \cdot x^{1/4}$

40. $(8x^{3/4})^{4/3}$

41. $\frac{6x^{3/4}}{3x^{1/2}}$

42. $\left(\frac{25x^{1/2}}{x^{1/6}}\right)^{1/2}$

LESSONS 80-86 Simplify. Write your answers in standard form.

43. $(6 + 5x^2 + 3x) + (2 - x - x^2)$

44. $(1 - 2x^2 - 4x) - (-3x + x^2 - 1)$

45. $(x - 6)(x + 3)$

46. $(2x - 3)(x^2 + 4x - 1)$

47. $(x^2 - 5x - 14) \div (x + 2)$

48. $(x^2 - 8) \div (x + 2)$

LESSON 101 Factoring Polynomials Completely

□ REFRESH YOUR SKILLS

1. Explain how to factor out the GCF from $3x^5 + 6x^4 + 18x$. Show your steps. Review Lesson 93 if needed.
2. Explain how to factor $3x^3 - 9x^2 + 2x - 6$ by grouping. Show your steps. Review Lesson 94 if needed.
3. Explain how to factor $4x^2 + 4x - 3$ by trial and error or by grouping. Show your steps. Review Lessons 97 and 98 if needed.

□ LEARN A SKILL: FACTORING POLYNOMIALS COMPLETELY

So far, you have learned various methods for factoring polynomials. To factor polynomials of any form, you first need to identify the type of polynomial and then decide which method(s) to use. Here is a general strategy. When you factor a polynomial, try these steps.

1. Factor out the GCF first (including -1 if the leading coefficient is negative).
2. If it is a special product, factor by following the pattern.
3. If it is a trinomial, factor by trial and error or by grouping.
4. If it has more than three terms, factor by grouping.
5. Check each factor to see if you can factor it further. If so, factor it again.

Let's see how to factor polynomials completely using the strategy above. We say a polynomial is factored completely when it cannot be factored any more.

→ **EXAMPLE** Factor completely.

$$2x^4 - 32$$

- ① $= 2(x^4 - 16)$
- ② $= 2(x^2 + 4)(x^2 - 4)$
- ③ $= 2(x^2 + 4)(x + 2)(x - 2)$

→ **EXAMPLE** Factor completely.

$$-x^5 + x^3 - 3x^2 + 3$$

- ① $= -(x^5 - x^3 + 3x^2 - 3)$
 $= -[x^3(x^2 - 1) + 3(x^2 - 1)]$
- ④ $= -(x^2 - 1)(x^3 + 3)$
- ⑤ $= -(x + 1)(x - 1)(x^3 + 3)$

→ **TRY IT** Factor completely.

4. $x^4 - 81$

5. $4x^3 - 100x$

→ **TRY IT** Factor completely.

6. $x^3 + x^2 - 9x - 9$

7. $-2x^3 - x^2 + 8x + 4$

□ **EXERCISE YOUR SKILLS**

Factor out the GCF. Check by multiplying.

8. $6x^2 + 10x - 6$

9. $25x^4 + 10x^3 + 15x^2$

10. $6x^2y - 15xy + 12y$

11. $4x^4y^2 - x^3y^3 + 8x^2y$

Factor using special product patterns. Check by multiplying.

12. $x^2 - 49$

13. $x^2 - 10x + 25$

14. $9x^2 + 12x + 4$

15. $4x^2 - 81$

Factor by trial and error or by grouping. Check by multiplying.

16. $x^2 + 3x - 40$

17. $x^2 - 9x + 18$

18. $12x^2 + 17x + 6$

19. $10x^2 - x - 3$

Factor by grouping. Check by multiplying.

20. $6x^3 - 18x^2 - x + 3$

21. $8x^3 - 40x^2 - 3x + 15$

22. $14x^3 + 7x^2 - 2x - 1$

23. $12x^3 + 8x^2 - 15x - 10$

Factor completely. Check by multiplying.

24. $8x^2 - 2x - 6$

25. $60x^2 - 9x - 6$

26. $x^3 - 49x$

27. $-8x^3 - 8x^2 - 2x$

28. $3x^4 - 48$

29. $x^3 + 4x^2 - 9x - 36$

30. $-10x^3 + 2x^2 - 15x + 3$

31. $9x^4 + 54x^3 + 15x + 90$

LESSON 127 Adding and Subtracting Rational Expressions

REFRESH YOUR SKILLS

Evaluate. Review Pre-Algebra if you have difficulty solving these problems.

1. $\frac{3}{5} + \frac{2}{5}$

2. $\frac{7}{9} - \frac{1}{9}$

3. $\frac{1}{3} + \frac{1}{2}$

4. $\frac{5}{6} - \frac{4}{9}$

Simplify. State any excluded values. Review Lesson 125 if needed.

5. $\frac{x^2 - 9}{x^2 - x - 6}$

6. $\frac{x^3 - 4x^2 - 5x}{x^3 - 5x^2 - x + 5}$

LEARN A SKILL: ADDING AND SUBTRACTING RATIONAL EXPRESSIONS WITH LIKE DENOMINATORS

To add or subtract rational expressions with like denominators, simply add or subtract their numerators and keep the common denominator. Then simplify as needed. Be sure to exclude from the domain the values that make the original denominators zero.

→ **EXAMPLE** Add. State any excluded values.

$$\frac{x^2 + 6x}{x + 3} + \frac{9}{x + 3} = \frac{x^2 + 6x + 9}{x + 3} = \frac{(x + 3)^2}{x + 3} = x + 3 \text{ for } x \neq -3$$

→ **TRY IT** Add or subtract. State any excluded values.

7. $\frac{x + 1}{2x + 3} + \frac{3x + 5}{2x + 3}$

8. $\frac{2x + 5}{x^2 + 3x - 10} - \frac{x + 7}{x^2 + 3x - 10}$

LEARN A SKILL: FINDING THE LCM OF POLYNOMIALS

To find the **least common multiple** (LCM) of two polynomials, first factor each polynomial completely and then find the product of the highest power of each factor.

→ **EXAMPLE** Find the LCM.

$$3x^2 - 27 \text{ and } x^2 + 6x + 9$$

↓ Factor each completely.

$$3(x + 3)(x - 3) \text{ and } (x + 3)^2$$

↓ Find the product of the highest power of each factor.

$$LCM = 3(x + 3)^2(x - 3)$$

→ **TRY IT** Find the LCM.

9. $x + 1$ and $x - 5$

10. $x^2 + x$ and $x^2 + 2x + 1$

11. $x^2 - 16$ and $3x^2 + 10x - 8$

□ **LEARN A SKILL: ADDING AND SUBTRACTING RATIONAL EXPRESSIONS WITH UNLIKE DENOMINATORS**

To add or subtract rational expressions with unlike denominators, first factor each denominator to find the **least common denominator** (LCD). Next, rewrite each fraction using the LCD as the denominator. Then add or subtract just as with rational expressions having like denominators. Remember, the LCD is the least common multiple (LCM) of the denominators.

→ **EXAMPLE** Subtract. State any excluded values.

$$\begin{aligned} \frac{x}{2x-4} - \frac{2}{x^2-2x} &= \frac{x}{2(x-2)} - \frac{2}{x(x-2)} = \frac{x \cdot x}{2(x-2) \cdot x} - \frac{2 \cdot 2}{x(x-2) \cdot 2} \quad \text{LCD} = 2x(x-2) \\ &= \frac{x^2-4}{2x(x-2)} = \frac{(x+2)(x-2)}{2x(x-2)} = \frac{x+2}{2x} \text{ for } x \neq 0, 2 \end{aligned}$$

→ **TRY IT** Add or subtract. State any excluded values.

12. $\frac{x+2}{x} - \frac{x+6}{x+4}$

13. $\frac{x^2-9x}{x^2+2x-3} + \frac{9}{x+3}$

□ **EXERCISE YOUR SKILLS**

Add or subtract. State any excluded values.

14. $\frac{x^2}{x+2} - \frac{4}{x+2}$

15. $\frac{x^3}{x^4-16} - \frac{4x}{x^4-16}$

16. $\frac{2x-5}{x^2+5x-14} + \frac{x-1}{x^2+5x-14}$

17. $\frac{x^2-5x}{2x^2+9x-5} - \frac{3x^2+8x-7}{2x^2+9x-5}$

18. $\frac{x+4}{x+5} - \frac{x-1}{x}$

19. $\frac{x^2+8}{x^2-2x-8} + \frac{2}{x+2}$

20. $\frac{x}{x^2+10x+25} - \frac{1}{x+5}$

21. $\frac{x-5}{x-6} + \frac{x-15}{x^2-3x-18}$

22. $\frac{1}{2x+1} + \frac{x-2}{2x^2+7x+3}$

23. $\frac{x^2+5x+1}{6x^2+x-1} + \frac{x}{2x+1}$

LESSON 152 Permutations and Combinations

REFRESH YOUR SKILLS

Solve. Assume repetition is not allowed. Review Lessons 147 and 148 if needed.

- How many 3-letter passwords can be formed from the letters A, B, and C?
- List all 3-letter passwords that can be formed from the letters A, B, and C.

LEARN A SKILL: IDENTIFYING PERMUTATIONS AND COMBINATIONS

A **permutation** is an arrangement of items in a particular order. A **combination** is an arrangement of items in which order does not matter. The difference is ordering.

EXAMPLE Permutation

How many 2-letter passwords can be formed from the letters A, B, and C?

The password AB is not same as the password BA. Order matters, so it is a permutation problem.

EXAMPLE Combination

How many teams of 2 players can be formed from the players A, B, and C?

The team AB is same as the team BA. Order does not matter, so it is a combination problem.

LEARN A SKILL: COUNTING PERMUTATIONS

You can use the counting principle to calculate the number of permutations.

EXAMPLE Counting permutations

In how many ways can you arrange 3 of the letters in the word MOUSE?

Number of permutations =

$$\binom{5 \text{ choices}}{1 \text{st letter}} \times \binom{4 \text{ choices}}{2 \text{nd letter}} \times \binom{3 \text{ choices}}{3 \text{rd letter}}$$

So, there are $5 \times 4 \times 3 = 60$ ways.

TRY IT Solve. Show your calculations.

- In how many ways can you arrange the letters in the word BIRD?
- In how many ways can you arrange 2 of the letters in the word DOLPHIN?

LEARN A SKILL: COUNTING COMBINATIONS

To find the number of combinations, take the permutations and divide by the repeats. Why?

Think about how many teams of 3 players can be formed from the players A, B, C, and D. We know there are $4 \times 3 \times 2 = 24$ possible permutations. However, because order doesn't matter, each group on the right should be counted as a single combination. Each group includes 6 permutations because there are $3 \times 2 \times 1 = 6$ ways to order 3 players. So, $24/6 = 4$ teams can be formed.

ABC, ACB
BAC, BCA
CAB, CBA

ABD, ADB
BAD, BDA
DAB, DBA

ACD, ADC
CAD, CDA
DAC, DCA

BCD, BDC
CBD, CDB
DBC, DCB

→ **EXAMPLE** Counting combinations

How many teams of 3 players can be formed from 5 players?

1. Find the number of permutations. There are $5 \times 4 \times 3 = 60$ ways to select 3 of the 5 players in order.
2. Find the number of repeats. There are $3 \times 2 \times 1 = 6$ ways to order 3 players, so 6 permutations should be counted as a single combination.
3. Divide the number of permutations by the number of repeats. So, $60/6 = 10$ teams can be formed.

→ **TRY IT** Solve. Show your calculations.

5. How many teams of 2 players can be formed from 3 players?
6. How many teams of 4 players can be formed from 9 players?
7. How many committees of 3 people can be formed from a group of 10 people?

□ **EXERCISE YOUR SKILLS**

Solve. Assume repetition is not allowed. Show your calculations. Use a calculator if necessary.

8. In how many ways can 4 people be seated in a row of 4 seats?
9. In how many ways can you arrange the letters of the word WHALE?
10. How many committees of 4 people can be formed from a group of 15?
11. In how many ways can a family of 6 people line up for a picture?
12. In how many ways can 2 colors be chosen from the 7 colors of the rainbow?
13. In how many ways can you arrange 3 of the letters in the word SPIDER?
14. In how many ways can you choose 4 dishes from a menu of 8 different dishes?
15. In how many ways can 3 cards be chosen from a standard deck of 52 cards?
16. In how many ways can 1st-, 2nd-, and 3rd-place prizes be awarded to 11 students?
17. How many matches will have to be played in a league of 10 teams if every team is to play every other team exactly once?
18. In how many ways can a president, a vice-president, a secretary, and a treasurer be chosen from a club of 20 members?
19. In how many ways can you draw the names of 3 raffle winners from a basket of 25 names if every winner gets the same prize?