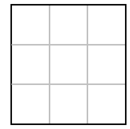
Lesson 29 Squares and Square Roots

A. The **square** of a number is the number times itself. The square of 3 is 9 because $3 \times 3 = 9$. The name "square" comes from the fact that the area of a square of side length x is equal to x^2, as shown on the right.



The area of the square
= length x width
$= 3 \times 3 = 3^2 = 9$

B. The **perfect squares**, also called **square numbers**, are the squares of the whole numbers. For example, 9 is a perfect square because $3^2 = 9$. The first five perfect squares are 1, 4, 9, 16, and 25.

C. A **square root** of a number n is a number x such that $x^2 = n$. It is just the opposite of squaring a number, as shown on the right. Every positive number has two square roots, one positive and one negative. Zero has exactly one square root, and negative numbers do not have square roots. For example, the square roots of 9 are both 3 and -3 since 3^2 is 9 and $(-3)^2$ is also 9. The square root symbol √, called a **radical**, indicates the non-negative square root. The number under the radical symbol is called the **radicand**.

3 squared is 9.

$3^2 = 9$

$\sqrt{9} = 3$

A square root of 9 is 3.

9 is the radicand of √9.

D. Find the square of each number.

2^2	6^2	0.5^2
8^2	9^2	1.1^2
11^2	12^2	1.3^2

E. Find the square root of each number. (Hint: Ask yourself, "What number multiplied by itself will produce this number?")

$\sqrt{1}$	$\sqrt{25}$	$\sqrt{49}$
$\sqrt{4}$	$\sqrt{64}$	$\sqrt{169}$
$\sqrt{36}$	$\sqrt{81}$	$\sqrt{225}$

F. Evaluate each expression using the order of operations. (Hint: Radical expressions such as square roots are treated the same as exponents in the order of operations.)

$2^3 - \sqrt{16} \times -4$

$\sqrt{4 + 7 \times 3} + \sqrt{64}$

$\left(3^3 - \sqrt{49}\right)^2 \div \sqrt{10^2}$

$\sqrt{144} \div \sqrt{36} - \sqrt{(2^2 \times 5^2)}$

Lesson 29 Practice

A. Find the square of each number.

1^2	3^2	0.2^2
4^2	5^2	0.6^2
7^2	8^2	1.2^2
10^2	13^2	1.5^2

B. Find the square root of each number.

$\sqrt{0}$	$\sqrt{4}$	$\sqrt{0.01}$
$\sqrt{9}$	$\sqrt{36}$	$\sqrt{0.49}$
$\sqrt{25}$	$\sqrt{81}$	$\sqrt{1.21}$
$\sqrt{144}$	$\sqrt{169}$	$\sqrt{1.96}$

C. Evaluate each expression using the order of operations.

$\sqrt{25} \times 4 - 3^2$

$(\sqrt{49} - 6)^3 \div 3^0 \times 3^2$

$\sqrt{144} \div (2^4 - \sqrt{16})$

$\sqrt{121} \times \sqrt{9} - \sqrt{8 + 7 \times 4}$

$\dfrac{(5+3)^2}{\sqrt{64}} - 9^2$

$\dfrac{\sqrt{225} - 3^2 \times \sqrt{81}}{\sqrt{196} - \sqrt{9}}$

D. Find the mystery numbers.

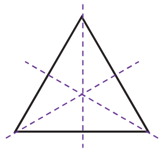
I am a 2-digit square number. The sum of my digits is my square root. What number am I? ____________

I am a 2-digit square number. My square root is a prime number. All of my digits are also prime numbers. What number am I? ____________

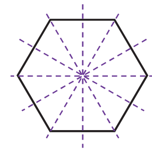
I am a 2-digit number divisible by 3 and 5 but not by 10. The sum of my digits is a square number. What number am I? ____________

Lesson 61 Symmetry

A. A figure has **reflection symmetry**, also known as **line symmetry** or **mirror symmetry**, if it can be divided in half by a line so that each half is a mirror image of the other. This dividing line is called a **line of symmetry**. Here are some examples.

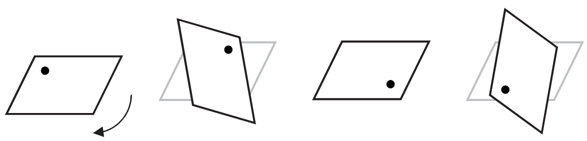


An equilateral triangle has 3 lines of symmetry, so it has reflection symmetry.

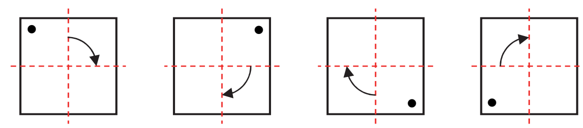


A regular hexagon has 6 lines of symmetry, so it has reflection symmetry.

B. A figure has **rotational symmetry** if it looks exactly the same more than once in one full turn (360°). The number of times the figure looks exactly the same is called the **order of rotational symmetry**. Here are some examples.



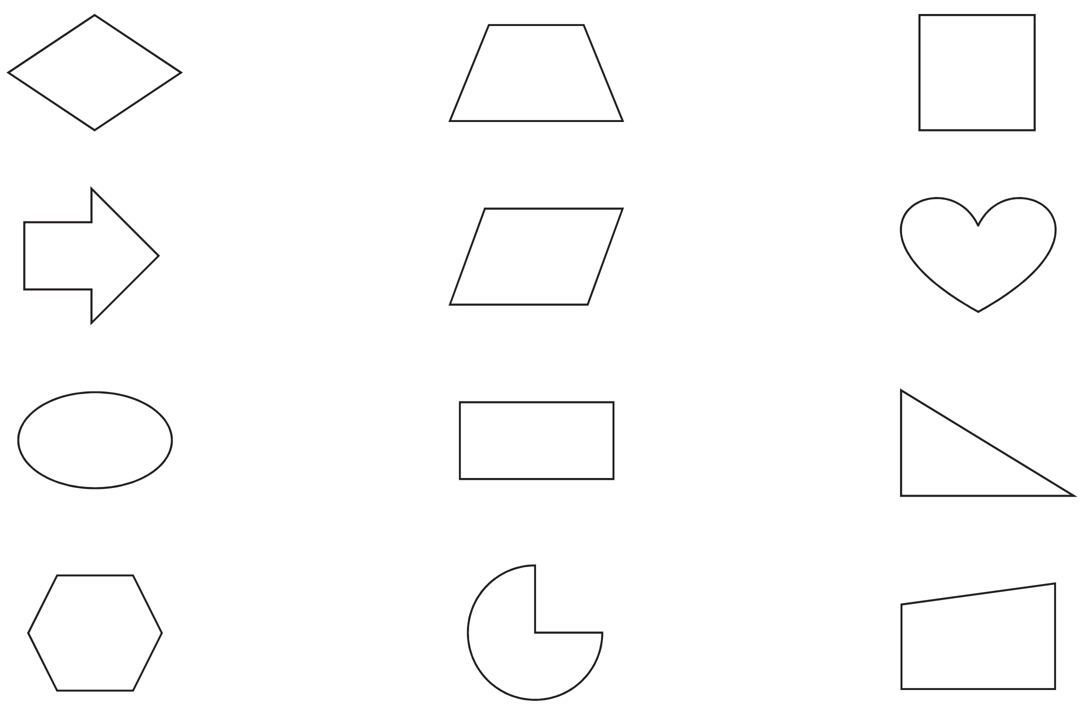
A parallelogram looks exactly the same every 180°, or 2 times in one full turn, so it has a rotational symmetry of order 2.



A square looks exactly the same every 90°, or 4 times in one full turn, so it has a rotational symmetry of order 4.

C. A figure with no rotational symmetry is sometimes said to have 'rotational symmetry of order 1' because it will look exactly the same only once after a complete turn. In other words, rotational symmetry of order 1 means that there is no symmetry at all.

D. Find the number of lines of symmetry and the order of rotational symmetry for each shape. Write 'none' if the shape has no rotational symmetry.



LESSON 61 Practice

A. Determine whether each statement is true or false.

True False A line of symmetry divides a shape into 2 congruent parts.

True False A parallelogram has more lines of symmetry than a kite.

True False A square has as many lines of symmetry as a rhombus.

True False A circle has infinitely many lines of symmetry.

True False A figure cannot have both reflection symmetry and rotational symmetry.

B. Draw lines of symmetry on each letter. Circle the letters that have rotational symmetry.

A D I T S W

H V C Y E X

Z M N O U B

C. Find the mystery numbers. Write your answers as decimals, if necessary.

Start with the exponent of x when $(x^5y^2)^{-4}$ is simplified. Subtract the number of acute angles in a square. What number do you have? ________

Start with the simplest radical form of $\sqrt{45}$. Take the radicand and multiply by the number of lines of symmetry in a rectangle. Add the largest prime factor of 56. What number do you have? ________

Start with the order of rotational symmetry of an equilateral triangle. Multiply by the exponent of 10 when 0.00014 is written in scientific notation. Add the square root of 196. What number do you have? ________

Start with the value of x that makes $6:9 = x:21$ true. Add the length of the hypotenuse of a right triangle whose legs are 5 and 12. Divide by the sum of all interior angles in a pentagon. What number do you have? ________

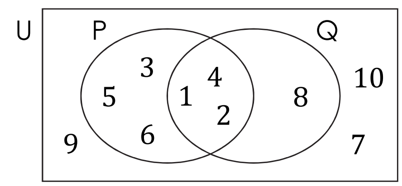
Lesson 80 Sets and Venn Diagrams

A. A **set** is a collection of things called **elements**. We can define a set by listing all of its elements in curly brackets. For example, the set of all numbers on a die is {1, 2, 3, 4, 5, 6}.

B. We usually use capital letters for sets. Let's say we denote the set of all numbers on a die as P and the set of factors of 8 as Q. Then we have P = {1, 2, 3, 4, 5, 6} and Q = {1, 2, 4, 8}.

C. A **universal set** is the set of all elements under consideration, usually denoted as U. Suppose we only consider the positive integers up to 10. That is, our U = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10}.

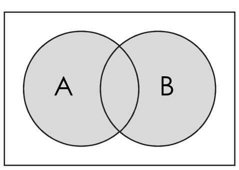
D. A **Venn diagram** is a diagram that shows the relationship among sets. In a Venn diagram, the universal set is generally drawn as a rectangle and all other sets are drawn as circles or ovals within the rectangle. On the right is the Venn diagram for our sets P, Q, and U.



E. Venn diagrams are useful when performing set operations. Below are four basic set operations. The shaded area in each diagram represents the result of the given operation.

Union

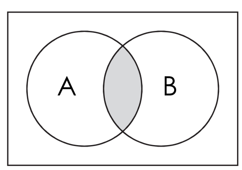
$A \cup B$



The set of all elements that are either in A or in B.

Intersection

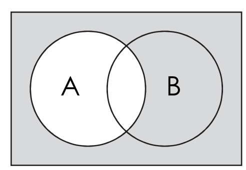
$A \cap B$



The set of elements that are both in A and in B.

Complement

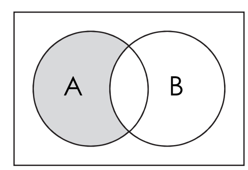
A^C



The set of elements that are in U but not in A

Difference

$A - B$



The set of elements that are in A but not in B

F. Use the Venn diagram of the sets P, Q, and U above to list the elements of each set.

$P \cap Q$

$Q - P$

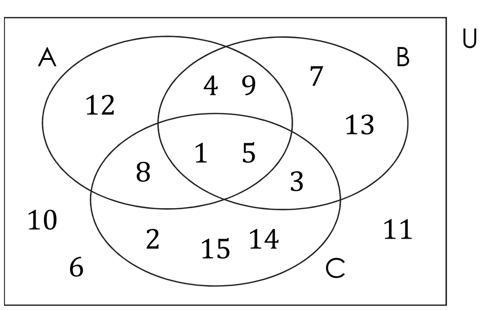
$U - P$

P^C

$P^C \cap Q$

$(P \cup Q)^C$

G. Use the given Venn diagram to list the elements of each set.



$A \cap B \cap C$

$(A \cup B \cup C)^C$

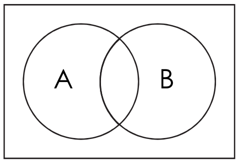
$(A \cap B) - C$

$A^C \cap (B \cup C)$

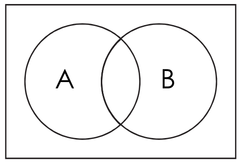
LESSON 80 Practice

A. Shade the region(s) representing the given set in each diagram.

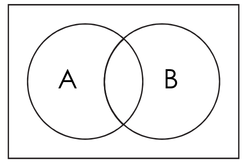
$(A \cup B)^C$



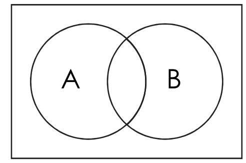
$(B - A)^C$



$A^C \cap B$



$B^C - A$



B. Let U = {1, 2, 3, 4, 5}, A = {1, 3, 5}, and B = {2, 3, 5}. List the elements of each set.

$A \cup B$

$A \cap B$

$(A \cup B)^C$

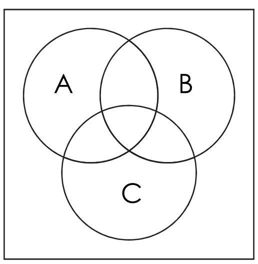
$(B - A)^C$

$A^C \cap B$

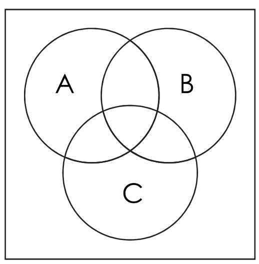
$B^C - A$

C. Shade the region(s) representing the given set in each diagram.

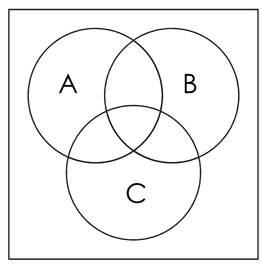
$(A \cup B \cup C)^C$



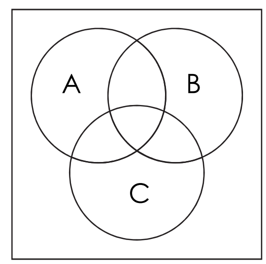
$(A \cup B) - C$



$C \cup (A - B)$



$(B \cup C)^C \cap A$



D. Let U = {all numbers on a die}, A = {prime numbers}, B = {odd numbers}, and C = {multiples of 3}. List the elements of each set.

$A \cup B \cup C$

$A \cap B \cap C$

$(A \cup B \cup C)^C$

$(A \cup B) - C$

$C \cup (A - B)$

$(B \cup C)^C \cap A$

E. Let U = {positive integers ≤ 15}, A = {integers divisible by 3}, B = {factors of 12}, C = {odd integers}. List the elements of each set.

$A \cap B \cap C$

$(A \cup B \cup C)^C$

$(A \cup B) - C$

$(B \cup C) \cap A$

$(A \cap B) - C$

$(A \cup B)^C \cap C$

LESSON 103 Absolute Value Equations

A. Define **absolute value** and **opposite numbers** in your own words. Give examples. Which values give an absolute value of 8? Review Lesson 14 if needed.

B. An **absolute value equation** is an equation that contains an absolute value expression. To solve an absolute value equation, rewrite the equation as two equations and solve each equation separately. The example on the right shows that there are two solutions to $|4x - 2| = 10$. Remember, it is always a good idea to check the solutions by substituting each into the original equation.

$$|4x - 2| = 10$$

Positive	Negative
$4x - 2 = 10$	$4x - 2 = -10$
$4x = 12$	$4x = -8$
$x = 3$	$x = -2$

C. Solve each equation. Simplify fractions.

$|x + 9| = 8$

$|x - 7| = 3$

$|3x| = 1.8$

$|4 - 2x| = 10$

$|-3x| - 7 = -5$

$|5x + 2| + 4 = 6$

$\dfrac{|x - 7|}{3} = 8$

$\left|\dfrac{4x + 7}{5}\right| = 15$

LESSON 103 Practice

Solve each equation. Simplify fractions.

$|x + 4| = 6$

$|x - 4| = 9$

$|0.9x| = 2.7$

$|-8.2x| = 41$

$|6x| - 8 = -3$

$|0.2x + 0.4| = 5.8$

$|9 - 3x\,| = 12$

$|7x + 6| - 3 = 9$

$\frac{|x - 6|}{5} = 7$

$\left|\frac{x}{9}\right| + 4 = 7$

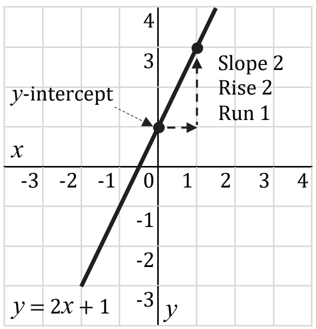
$\left|\frac{2x - 5}{3}\right| = 1.2$

$\left|\frac{8 - 4x}{5}\right| - 9 = -5$

LESSON 116 Graphing Linear Equations in Slope-Intercept Form

A. Get graph paper (provided in Appendix A). Graph the equation $y = 2x + 1$ by making a table of ordered pairs and plotting the points. Review Lesson 114 if needed.

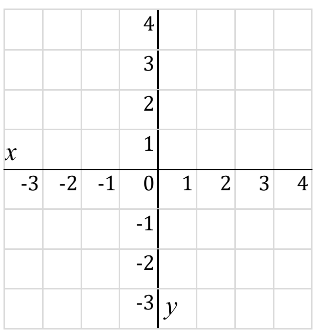
B. When solved for y, all linear equations have the form $y = mx + b$. This is called **slope-intercept form** because it tells us the slope m and the y-intercept b of the line. The ***y*-intercept** is the point where the line intersects the y-axis. For example, without making a table, we can say that the graph of $y = 2x + 1$ is a line with a slope of 2 and a y-intercept of 1. Below are the steps to graph $y = 2x + 1$ using the slope and y-intercept. Compare the line below with your graph.



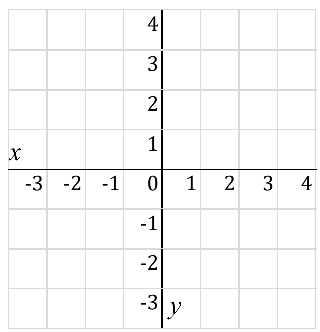
To graph the equation $y = 2x + 1$:

1. Use the y-intercept to plot the first point. The y-intercept is 1, so we first plot (0, 1).
2. From the first point, use the slope to find the second point and plot it. The slope is 2, so we move to the right 1 unit and up 2 units from (0, 1) to reach (1, 3) and plot it.
3. Draw a line through the two points. The line passing through (0, 1) and (1, 3) represents $y = 2x + 1$.

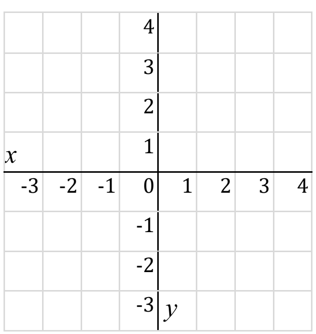
C. Find the slope and y-intercept for each equation. Then graph the line.



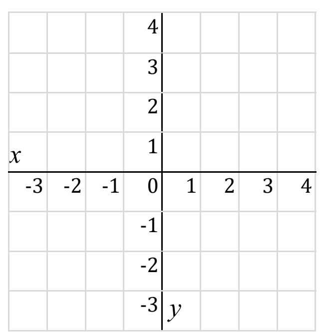
$y = x + 2$



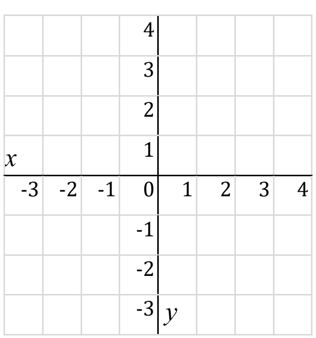
$y = 2x - 3$



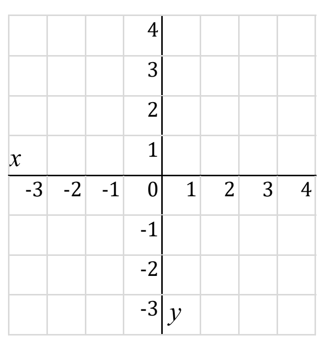
$y = \frac{2}{3}x - 2$



$y = -\frac{1}{2}x + 1$



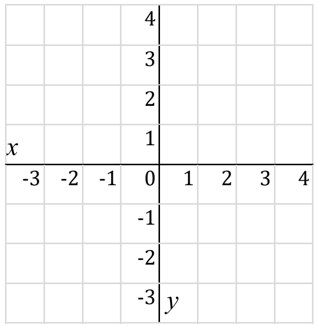
$y = -x + 3$



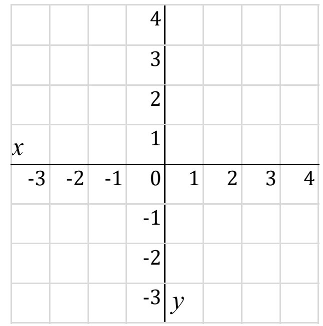
$y = \frac{3}{4}x - 1$

LESSON 116 Practice

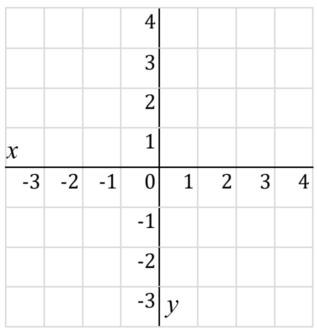
A. Find the slope and y-intercept for each equation. Then graph the line.



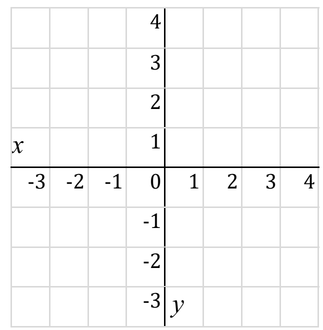
$y = \text{-}x - 1$



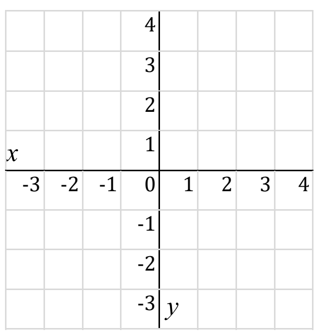
$y = \frac{1}{2}x + 3$



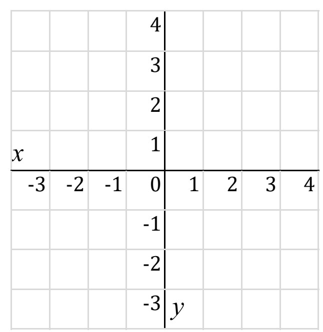
$y = 4x$



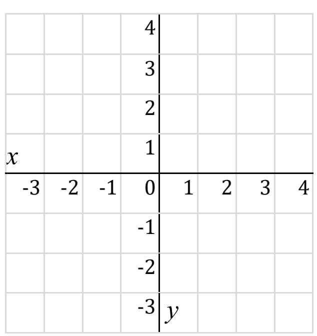
$y = \text{-}\frac{2}{3}x - 2$



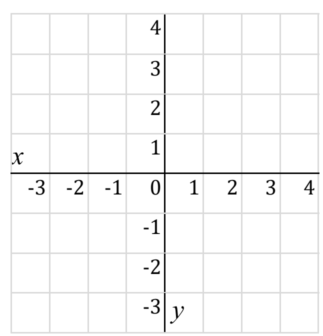
$y = \text{-}2x + 2$



$y = x - 3$



$y = \frac{1}{4}x - 1$



$y = \frac{3}{2}x - 2$

B. Solve each problem.

A line passes through the points (1, 2) and (0, 7). What is its equation in slope-intercept form? ________________

A right triangle is formed by the x-axis, the y-axis, and the graph of the equation $y = \text{-}2x + 4$. Find the length of the hypotenuse of the right triangle. Write your answer in simplest radical form. ________________

Lesson 142 Catch up and Review!

A. Catch up if you are behind. You can go back and redo a lesson that caused you trouble, and you can use the activity below to help you review.

B. Which of the following statements are true? Select all that apply.

Lesson 95 The expression $x^2 + 2xy + y^2$ evaluates to 16 when $x = 5$ and $y = 1$.

Lesson 96 The expression $5(x + 2) - 2(3x + 2)$ simplifies to $-x + 6$.

Lesson 98 The equations $x + 2 = -1$ and $5x = -15$ have the same solution.

Lesson 99 The equations $2x + 7 = 1$ and $-2x + 3 = -3$ have the same solution.

Lesson 102 The equations $5x - 9 = x - 1$ and $5(x - 1) = 5$ have the same solution.

Lesson 103 The equation $|x + 4| = 7$ has one positive and one negative solution.

Lesson 106 The equation $3x + 2y = 5x - 4$ becomes $y = x - 2$ when solved for y.

Lesson 109 The inequality $x + 6 > 9$ has the solution set $x < 3$.

Lesson 110 The inequalities $-3x \leq 15$ and $2x + 3 \leq 7$ have the same solution sets.

Lesson 115 The slope of a line passing through (2, -4) and (-1, 5) is -3.

Lesson 116 The graph of $y = 4x + 1$ is a line with a slope of 4 and a y-intercept of 1.

Lesson 117 The graph of $x - 2y = 0$ is a line with a slope of 2 and a y-intercept of 0.

Lesson 118 The graph of $4x - 3y = 12$ has an x-intercept of 3 and a y-intercept of -4.

Lesson 119 The graph of $y = 3$ is a vertical line with a y-intercept of 3.

Lesson 122 The graphs of $x - 2y = 2$ and $2x + y = 4$ intersect at (-2, 0)

Lesson 123 (4, -2) is the solution to the linear system $2x + 5y = -2$ and $3x + 4y = 4$.

Lesson 124 The linear system $3x - 3y = 9$ and $y = x + 4$ has no solution.

Lesson 128 The equation of a line passing through (-2, 5) with slope 4 is $y = 4x + 3$.

Lesson 129 The equation of a line passing through (1, -4) and (3, 0) is $y = 2x - 6$.

Lesson 136 The relation {(-1, -2), (0, 1), (1, 4), (2, 7)} represents a linear function.

Lesson 139 The 15^{th} term of the arithmetic sequence 8, 5, 2, -1, -4, … is -34.

Lesson 142 Practice

A. Solve each equation.

$2x + 4 = 10$

$3 - 4x = -13$

$29 = x - 6x + 4$

$5x - 4 = -9$

$8 + 4x = -15$

$19 = 2x - 7x + 7$

B. Solve each inequality.

$x + 9 > 23$

$-5x \leq 75$

$-15 < 8(x - 5) + 33$

$2(2x + 1) > 10$

$9 \leq 2(8 - x)$

$2.7x - 4.2x \geq 4.5$

C. Solve each problem.

Jessica put 22 feet of fencing around her rectangular garden with a width of 5 feet. What is the length of her garden? ____________________

Maria bought some pencils at $0.30 each and 4 erasers at $0.15 each. The total cost was $3.30. How many pencils did Maria buy? ____________________

A right triangle has a base of 8 cm. Its area is at least 16 cm^2 and at most 36 cm^2. What are the smallest possible height and largest possible height of the triangle? ____________________

A cuboid has a width of 4 cm and a length of 6 cm. Its volume is at least 72 cm^3 and at most 192 cm^3. What is the difference between the largest possible height and the smallest possible height? ____________________