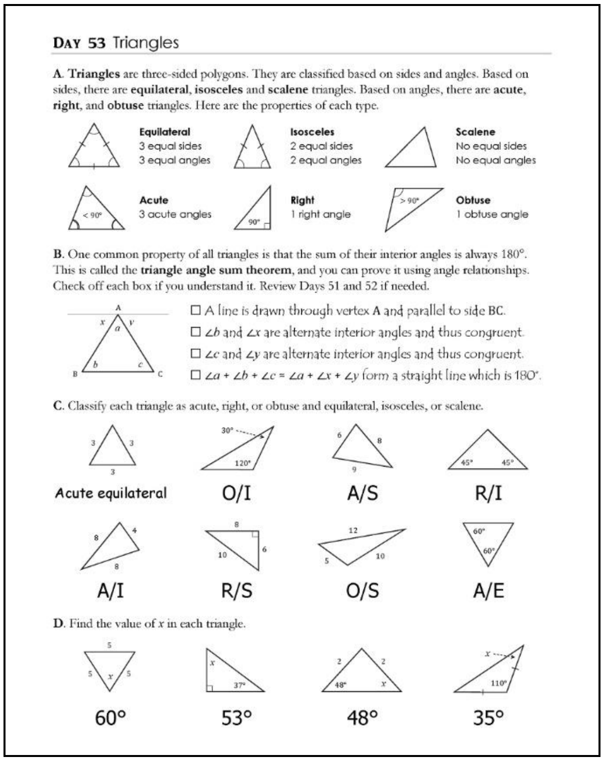
Day 53



Day 53 Triangles

A. **Triangles** are three-sided polygons. They are classified based on sides and angles. Based on sides, there are **equilateral**, **isosceles** and **scalene** triangles. Based on angles, there are **acute**, **right**, and **obtuse** triangles. Here are the properties of each type.

Equilateral 3 equal sides 3 equal angles

Isosceles 2 equal sides 2 equal angles

Scalene No equal sides No equal angles

Acute 3 acute angles

Right 1 right angle

Obtuse 1 obtuse angle

B. One common property of all triangles is that the sum of their interior angles is always 180°. This is called the **triangle angle sum theorem**, and you can prove it using angle relationships. Check off each box if you understand it. Review Days 51 and 52 if needed.

- ☐ A line is drawn through vertex A and parallel to side BC.
- ☐ $\angle b$ and $\angle x$ are alternate interior angles and thus congruent.
- ☐ $\angle c$ and $\angle y$ are alternate interior angles and thus congruent.
- ☐ $\angle a + \angle b + \angle c = \angle a + \angle x + \angle y$ form a straight line which is 180°.

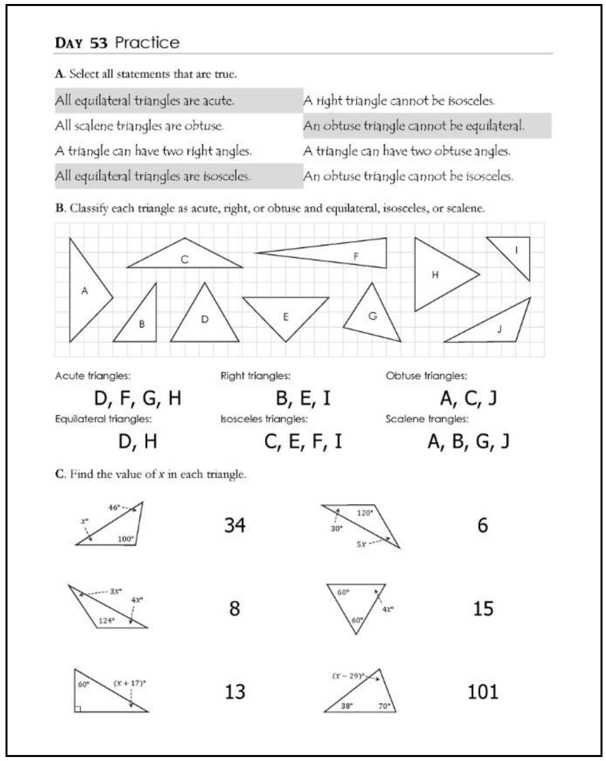
C. Classify each triangle as acute, right, or obtuse and equilateral, isosceles, or scalene.

Acute equilateral | O/I | A/S | R/I

A/I | R/S | O/S | A/E

D. Find the value of x in each triangle.

60° | 53° | 48° | 35°



Day 53 Practice

A. Select all statements that are true.

All equilateral triangles are acute.	A right triangle cannot be isosceles.
All scalene triangles are obtuse.	An obtuse triangle cannot be equilateral.
A triangle can have two right angles.	A triangle can have two obtuse angles.
All equilateral triangles are isosceles.	An obtuse triangle cannot be isosceles.

B. Classify each triangle as acute, right, or obtuse and equilateral, isosceles, or scalene.

Acute triangles: D, F, G, H	Right triangles: B, E, I	Obtuse triangles: A, C, J
Equilateral triangles: D, H	Isosceles triangles: C, E, F, I	Scalene trangles: A, B, G, J

C. Find the value of x in each triangle.

34 | 6

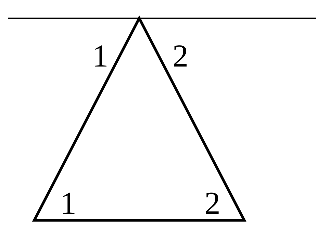
8 | 15

13 | 101

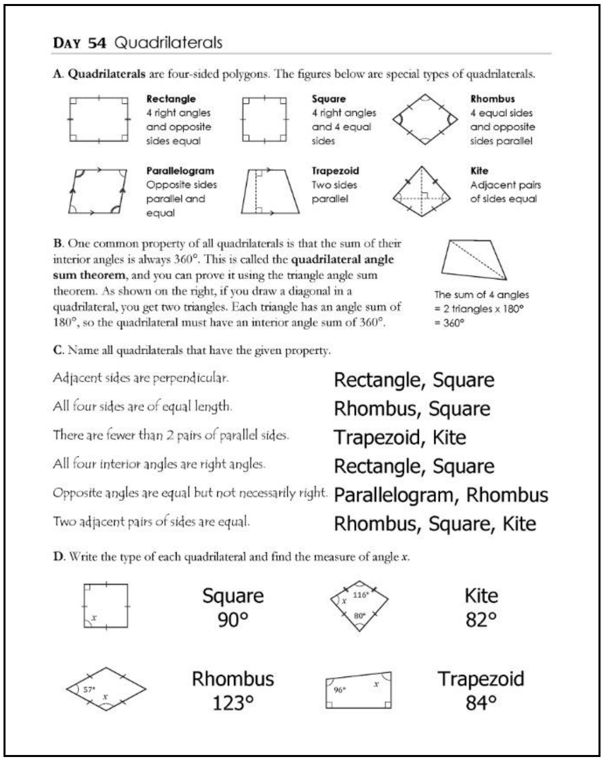
Day 53 Lesson – Triangles

They should already be familiar with the different types of angles and triangles. The angles inside the triangle added together come to 180 degrees.

You can draw a line parallel to the base of the triangle to get more information about the angles and how much they are worth. The new line and the base of the triangle are parallel lines. You can draw the triangle lines to cross the parallel lines. It will create those exterior and interior angles that are equal. I labeled equal angles with the same number.



Day 54



DAY 54 Quadrilaterals

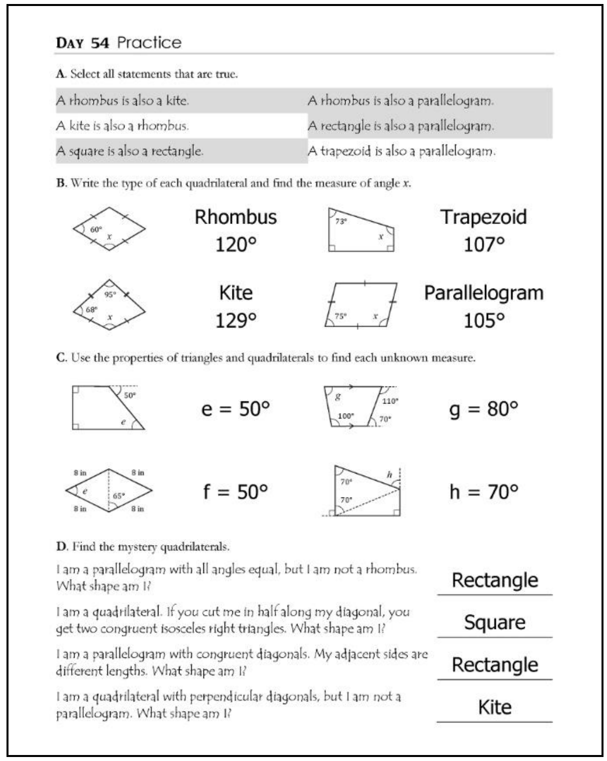
A. **Quadrilaterals** are four-sided polygons. The figures below are special types of quadrilaterals.

B. One common property of all quadrilaterals is that the sum of their interior angles is always 360°. This is called the **quadrilateral angle sum theorem**, and you can prove it using the triangle angle sum theorem. As shown on the right, if you draw a diagonal in a quadrilateral, you get two triangles. Each triangle has an angle sum of 180°, so the quadrilateral must have an interior angle sum of 360°.

C. Name all quadrilaterals that have the given property.

Adjacent sides are perpendicular.	Rectangle, Square
All four sides are of equal length.	Rhombus, Square
There are fewer than 2 pairs of parallel sides.	Trapezoid, Kite
All four interior angles are right angles.	Rectangle, Square
Opposite angles are equal but not necessarily right.	Parallelogram, Rhombus
Two adjacent pairs of sides are equal.	Rhombus, Square, Kite

D. Write the type of each quadrilateral and find the measure of angle *x*.



DAY 54 Practice

A. Select all statements that are true.

A rhombus is also a kite.	A rhombus is also a parallelogram.
A kite is also a rhombus.	A rectangle is also a parallelogram.
A square is also a rectangle.	A trapezoid is also a parallelogram.

B. Write the type of each quadrilateral and find the measure of angle *x*.

C. Use the properties of triangles and quadrilaterals to find each unknown measure.

D. Find the mystery quadrilaterals.

I am a parallelogram with all angles equal, but I am not a rhombus. What shape am I?	Rectangle
I am a quadrilateral. If you cut me in half along my diagonal, you get two congruent isosceles right triangles. What shape am I?	Square
I am a parallelogram with congruent diagonals. My adjacent sides are different lengths. What shape am I?	Rectangle
I am a quadrilateral with perpendicular diagonals, but I am not a parallelogram. What shape am I?	Kite

Day 54 Lesson – Quadrilaterals

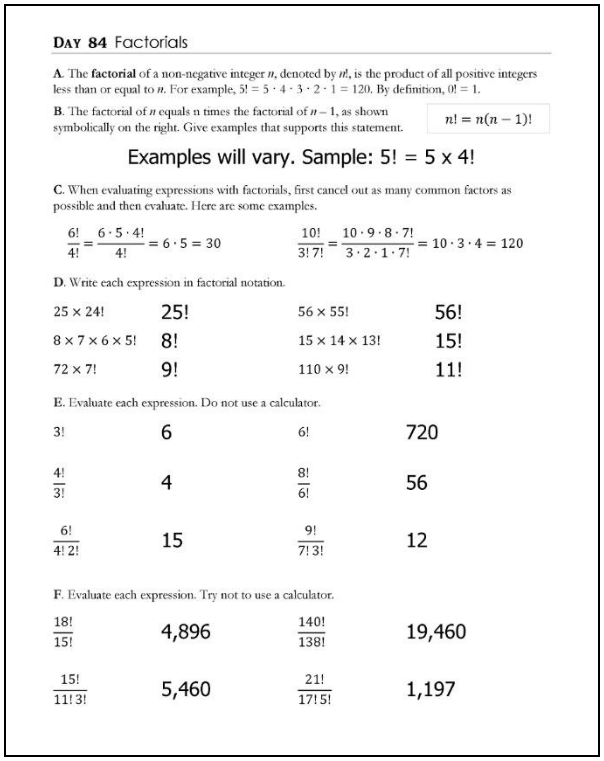
There are a bunch of pictures on this page. They are quadrilaterals. “Quad” refers to four. These all have four sides. The differences are in the angle and side measurements.

The angles inside of a triangle add up to 180 degrees, like a straight line. A square or a rectangle is made up of two triangles. Can you picture that? Just draw a line from one corner to the other. Two triangles are 180 degrees plus 180 degrees. That’s 360 degrees, just like a circle.

They will only be find a few angles on the page. The squares mean right angles, 90 degrees. The little dash lines mean that those sides are equal. A set of double dash lines means those are equal.

Mostly, they are identifying the different types of quadrilaterals. In Part C, there can be several answers, so they need to think beyond just finding one answer. They need to rule in or out each quadrilateral.

Day 84



DAY 84 Factorials

A. The **factorial** of a non-negative integer n, denoted by $n!$, is the product of all positive integers less than or equal to n. For example, $5! = 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 120$. By definition, $0! = 1$.

B. The factorial of n equals n times the factorial of $n - 1$, as shown symbolically on the right. Give examples that supports this statement.

$n! = n(n-1)!$

Examples will vary. Sample: 5! = 5 x 4!

C. When evaluating expressions with factorials, first cancel out as many common factors as possible and then evaluate. Here are some examples.

$\frac{6!}{4!} = \frac{6 \cdot 5 \cdot 4!}{4!} = 6 \cdot 5 = 30$ $\qquad$ $\frac{10!}{3!\,7!} = \frac{10 \cdot 9 \cdot 8 \cdot 7!}{3 \cdot 2 \cdot 1 \cdot 7!} = 10 \cdot 3 \cdot 4 = 120$

D. Write each expression in factorial notation.

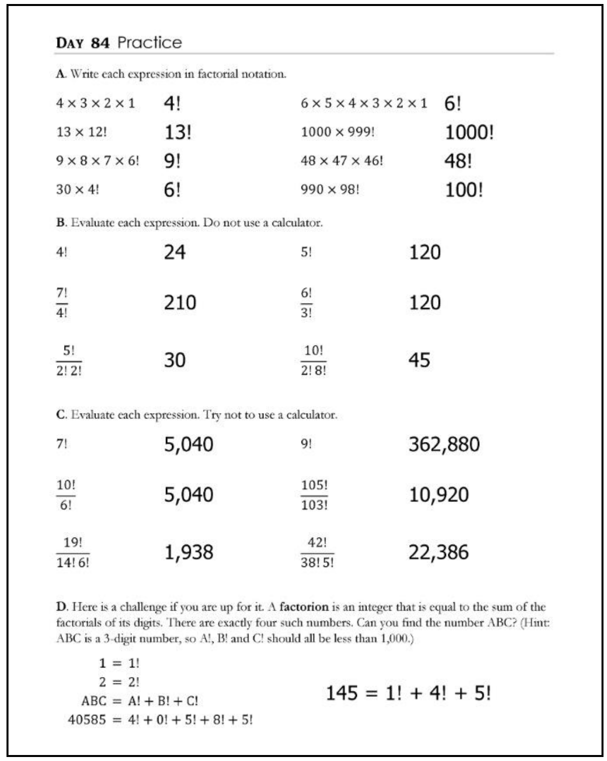
$25 \times 24!$	25!	$56 \times 55!$	56!
$8 \times 7 \times 6 \times 5!$	8!	$15 \times 14 \times 13!$	15!
$72 \times 7!$	9!	$110 \times 9!$	11!

E. Evaluate each expression. Do not use a calculator.

$3!$	6	$6!$	720
$\frac{4!}{3!}$	4	$\frac{8!}{6!}$	56
$\frac{6!}{4!\,2!}$	15	$\frac{9!}{7!\,3!}$	12

F. Evaluate each expression. Try not to use a calculator.

$\frac{18!}{15!}$	4,896	$\frac{140!}{138!}$	19,460
$\frac{15!}{11!\,3!}$	5,460	$\frac{21!}{17!\,5!}$	1,197



DAY 84 Practice

A. Write each expression in factorial notation.

$4 \times 3 \times 2 \times 1$	4!	$6 \times 5 \times 4 \times 3 \times 2 \times 1$	6!
$13 \times 12!$	13!	$1000 \times 999!$	1000!
$9 \times 8 \times 7 \times 6!$	9!	$48 \times 47 \times 46!$	48!
$30 \times 4!$	6!	$990 \times 98!$	100!

B. Evaluate each expression. Do not use a calculator.

$4!$	24	$5!$	120
$\frac{7!}{4!}$	210	$\frac{6!}{3!}$	120
$\frac{5!}{2!\,2!}$	30	$\frac{10!}{2!\,8!}$	45

C. Evaluate each expression. Try not to use a calculator.

$7!$	5,040	$9!$	362,880
$\frac{10!}{6!}$	5,040	$\frac{105!}{103!}$	10,920
$\frac{19!}{14!\,6!}$	1,938	$\frac{42!}{38!\,5!}$	22,386

D. Here is a challenge if you are up for it. A **factorion** is an integer that is equal to the sum of the factorials of its digits. There are exactly four such numbers. Can you find the number ABC? (Hint: ABC is a 3-digit number, so A!, B! and C! should all be less than 1,000.)

1 = 1!
2 = 2!
ABC = A! + B! + C!
40585 = 4! + 0! + 5! + 8! + 5!

145 = 1! + 4! + 5!

Day 84 Lesson – Factorials

These are fun. They are shown by an exclamation point. What's not fun about exclamation points!? The factorial of 3 is 3 times 2 times 1. Of course, anything times 1 is just itself, so that part isn't really necessary.

n! = n(n-1)! because 7! = 7 x 6!

7 x 6 x 5 x 4 x 3 x 2 x 1 = 7 x 6 x 5 x 4 x 3 x 2 x 1

What I just wrote out above is what Part B and Part D are about. When they get to 72 x 7! and it doesn't seem like the others, it is! Encourage them to think about what every other answer was and how this has to be something similar. (Hint: 72 = 8 x 9)

Part C's examples go with Part E. They should cancel out to solve. 3x/x is just 3. You can cross out the X and not worry about it. It cancels out because x/x = 1 and multiplying by 1 doesn't change the answer. We call that cancelling out. We can just ignore it. so 7!/6! = $\frac{7 \cdot 6!}{6!}$ = 7 The 6! cancels out.

Day 85

DAY 85 Permutations and Combinations

A. A **permutation** is an arrangement of items in a particular order. Suppose a box contains three letters X, Y, and Z. You randomly draw the first letter from the box, set it aside, and then draw the second letter. In how many different ways can you draw two letters? Since you draw them in order, you have six different ways: XY, XZ, YX, YZ, ZX, and ZY. So we say that there are six possible permutations to choose two letters from X, Y, and Z.

B. A **combination** is an arrangement of items in which order does not matter. Suppose, in the example above, you draw two letters at once so that there is no order. In how many different ways can you draw two letters this time? Since order doesn't matter, you have three different ways: XY (or YX), XZ (or ZX), and YZ (or ZY). So we say that there are three possible combinations to choose two letters from X, Y, and Z.

C. You can use the **fundamental counting principle** to calculate the number of permutations or combinations for a given situation. For example, how many permutations and combinations are there when three letter are selected at random from the set {A, B, C, D}? There are four ways to pick the first letter, three ways to pick the second letter, and two ways to pick the third letter. So we have $4 \times 3 \times 2 = 24$ permutations. To get the number of combinations, take the number of permutations and divide by the repeats. ABC, ACB, BAC, BCA, CAB, and CBA are treated as different permutations, but they all should be counted as a single combination. Since three letters can be arranged in $3 \times 2 \times 1 = 6$ ways, there are six repeats for each permutation. So we have $24/6 = 4$ combinations. Review Day 83 if needed.

D. Determine whether each situation involves a permutation or a combination. Then answer the question. Assume repetition is not allowed. Show your work. You may use a calculator.

In how many different ways can 4 people be seated in a row of 4 seats?

P 4 x 3 x 2 x 1 = 24

In how many different ways can you arrange the letters of the word MOUSE?

P 5 x 4 x 3 x 2 x 1 = 120

How many different committees of 4 people can be chosen from a group of 15?

C (15 x 14 x 13 x 12)/(4 x 3 x 2 x 1) = 1,365

In how many different ways can 1st-, 2nd-, and 3rd-place prizes be awarded to 11 contestants?

P 11 x 10 x 9 = 990

In how many different ways can you draw the names of 3 raffle winners from a basket of 25 names if every winner gets the same prize?

C (25 x 24 x 23)/(3 x 2 x 1) = 2,300

DAY 85 Practice

A. List all the permutations of the numbers 1, 2, and 3.

123, 132, 213, 231, 312, 321

B. List all the permutations of two numbers chosen without repetition from the set {1, 2, 3, 4}.

12, 21, 13, 31, 14, 41, 23, 32, 24, 42, 34, 43

C. List all the combinations of two numbers chosen without repetition from the set {1, 2, 3, 4}.

12, 13, 14, 23, 24, 34

D. List all the combinations of three numbers chosen without repetition from the set {1, 2, 3, 4}.

123, 124, 134, 234

E. Determine whether each situation involves a permutation or a combination. Then answer the question. Assume repetition is not allowed. Show your work. You may use a calculator.

In how many different ways can a family of 6 people line up for a picture?

P 6 x 5 x 4 x 3 x 2 x 1 = 720

In how many different ways can 3 cards be chosen from a deck of 52 cards?

C (52 x 51 x 50)/(3 x 2 x 1) = 22,100

How many different 4-letter codes can you make from the letters in the word GRAPHS?

P 6 x 5 x 4 x 3 = 360

In how many different ways can you choose 4 dishes from a menu of 8 different dishes?

C (8 x 7 x 6 x 5)/(4 x 3 x 2 x 1) = 70

How many different committees of 5 people can be formed from a group of 10 people?

C (10 x 9 x 8 x 7 x 6)/(5 x 4 x 3 x 2 x 1) = 252

In how many different ways can a president, a vice-president, a secretary, and a treasurer be chosen from a club of 20 members?

P 20 x 19 x 18 x 17 = 116,280

Day 85 Lesson – Permutations and Combinations

Permutations is a type of possible outcome. Let's say two people were chosen at random to win $100 and $50 gift certificates. They were going to choose by drawing names from a hat. If there were five names in the hat, the first time one is drawn, there are 5 possible outcomes. Then another name is drawn; this time there are only 4 possible outcomes. There were 5 x 4 possible outcomes.

In combinations, the order doesn't matter. 123 and 321 are counted as one possible outcome. Let's use the example above again. This time they are drawing two names to be assigned to a task. Whether Paul and Obed or Obed and Paul are picked, it makes no difference, those two would be assigned the task. Obed and Paul as the two choices or Paul and Obed as the two choices count as one possible outcome. We found out the number of permutations in this scenario is 5 x 4 or 20 possible outcomes. Then we divide that by the number of repeated outcomes, 2. (This is different from the number of outcomes. Once Paul and Obed are chosen, how many other possibilities are there for those two names to come up, just one other time, if Obed is chosen first and then Paul.)

Permutations / repeats = 20 / 2 = 10 This is the total number of <u>combinations</u>.

If they are getting stuck, have them write out/draw examples. You can even act out the first one with the four chairs.

Day 86

DAY 86 Permutations and Factorials

A. Explain what **factorial** is and how it works. Give examples. Review Day 84 if needed.

See Day 84, Part A.

B. Define **permutation** and **combination** in your own words. Give examples that illustrate the difference between the two. Review Day 85 if needed.

See Day 85, Parts A and B.

C. Permutation problems can be solved using the counting principle or the permutation formula. See the formula on the right.

$$\text{Number of permutations of } n \text{ things taken } r \text{ at a time} = \frac{n!}{(n-r)!}$$

D. Let's see an example. In how many ways can you arrange 3 letters from the word ORANGE? Below are two ways to calculate the number of permutations of 6 letters taken 3 at a time.

Using the counting principle: $6 \times 5 \times 4 = 120$

Using the formula: $\frac{6!}{(6-3)!} = \frac{7!}{3!} = 6 \times 5 \times 4 = 120$

E. Determine whether each situation involves a permutation or a combination. Then answer the question. Assume repetition is not allowed. Show your work. You may use a calculator.

In how many different ways can you arrange 5 different books on a shelf?

P 5! = 120

In how many different ways can 5 students be chosen to represent a class of 22 students?

C 22!/17! divided by 5! = 26,334

How many different teams of 5 players can be formed from a group of 8 boys and 7 girls?

C 15!/10! divided by 5! = 3,003

In how many different ways can you arrange the letters of the word COMPUTER taking 4 letters at a time?

P 8!/4! = 1, 680

In how many different ways can a president, a vice-president, and a treasurer be elected from a class of 30 students?

P 30!/27! = 24,360

DAY 86 Practice

A. Evaluate each expression. Do not use a calculator.

$\frac{7!}{5!}$	42	$\frac{9!}{8!}$	9
$\frac{10!}{5!\,5!}$	252	$\frac{12!}{9!\,4!}$	55
$\frac{18!}{6!\,14!}$	102	$\frac{25!}{22!\,5!}$	115

B. Determine whether each situation involves a permutation or a combination. Then answer the question. Assume repetition is not allowed. Show your work. You may use a calculator.

In how many different ways can you arrange the letters in the word KEYBOARD?

P 8! = 40,320

How many different 3-letter codes can you make from the letters in the word KEYBOARD?

P 8!/5! = 336

In how many ways can you choose 4 toppings from 10 different toppings for your pizza?

C 10!/6! divided by 4! = 210

In how many different ways can you choose 6 desserts from a tray of 12 different desserts?

C 12!/6! divided by 6! = 924

In how many different ways can 1st-, 2nd-, and 3rd-place prizes be awarded to 18 contestants?

P 18!/15! = 4,896

In how many different ways can you make a 4-letter passcode using the alphabet?

P 26!/22! = 358,800

In how many different ways can you draw the names of 2 raffle winners from a basket of 40 names if every winner gets the same prize?

C 40!/38! divided by 2! = 780

Day 86 Lesson – Permutations and Factorials

When we are drawing names from a hat, from 5 down to 1, the outcome possibilities are 5 x 4 x 3 x 2 x 1 or 5!. Can you see the relationship between the counting principle and factorials?

If we were just picking out three names, the number of outcomes would be 5 x 4 x 3 = 60. That would NOT be equal to 5! because it doesn't have times two there. Here's the formula for outcomes in that case.

$$\frac{5!}{(5-3)!} = \frac{5!}{2!} = 5 \times 4 \times 3 = 60$$

Part E

first one, arranging five books: 5!

five students out of twenty-two: 22!/(22-5)!

five players out of fifteen: 15!/(15-5)!