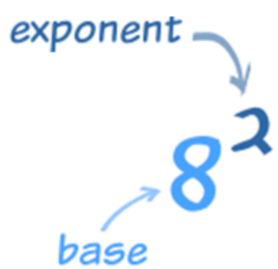
Course Name: **MS Math 8**
Module Name: **Exponents**

Lesson Topic – Exponential Expressions

When you need to write repeated addition, you typically use multiplication, right? How, then, would you write repeated multiplication?

Writing repeated multiplication is done by using exponents. A power consists of a base and an exponent. The base is the factor that is being multiplied by itself and the exponent is the number that tells you how many times that factor is used.

Let's take a look. This is a power. You can see the base and the exponent are labeled.



This means 8 (base) is listed as a factor 2 (exponent) times. In other words, $8^2 = 8•8$, or "8 squared."

Let's look at another one. How could we expand the power 6^4? Well, since the base is 6, that means we'll use 6 as our factor. Since the exponent is 4, we'll use that base as a factor 4 times. In other words, $6^4 = 6•6•6•6$.

Now let's look at this from the other direction. How can we rewrite 2•2•2•2•2 as a power? The factor we are multiplying by itself is 2, so 2 will be our base. How many times is the 2 listed as a factor? That's right, 5, so 5 will be our exponent. $2•2•2•2•2 = 2^5$.

When looking at problems like this, you need to be careful not to make a common mistake. If you multiply out 2^3, what do you get? If you said 6, let's think about that again. $2^3 = 2•2•2 = 4•2=8$. Always be careful to NOT multiply the base times the exponent. Remember, you are multiplying the base by itself, not by the exponent.

Now, let's take this a step further. What do we do if we want to perform operations with powers?

Let's investigate a bit. If we want to multiply $4^3 \bullet 4^2$, we could list these all out and see how many factors of 4 we have.

$4^3 \bullet 4^2 = 4 \bullet 4 \bullet 4 \bullet 4 \bullet 4 = 4^5$

You can see from this example that if we list all the factors of 4 and then count them, we have 4^5. This is actually leads us to an rule regarding multiplication of powers…

If you are multiplying like bases, then you may keep the base and add the exponents. This is essentially what we did above with $4^3 \bullet 4^2 = 4 \bullet 4 \bullet 4 \bullet 4 \bullet 4 = 4^5$.

We can rewrite this as a general rule as follows:

$\mathbf{a^m + a^n = a^{m+n}}$

Another operation where we can observe a pattern that leads us to a general rule is with division of powers.

$$\frac{x^5}{x^2} = \frac{\cancel{x} \bullet \cancel{x} \bullet x \bullet x \bullet x}{\cancel{x} \bullet \cancel{x}} = x \bullet x \bullet x = x^3$$

While writing it out like this is acceptable, it can get extremely tedious if your exponent is 100!! So, we use properties to help us. Here we can see that if we have the same base and we are dividing powers, we can keep the base and subtract the exponents. In general, we can say the following.

$$\mathbf{\frac{a^m}{a^n} = a^{m-n}}$$

What happens if we get a negative exponent? Typically, it is more mathematically correct to write a power with all positive exponents. We can use the property just mentioned to help us see what to do with negative exponents. Let's take our example from before, but this time let's read it as its reciprocal.

$$\frac{x^2}{x^5}$$

Based on what we just mentioned, we should be able to keep the base and subtract the exponents. That would give us x^{2-5}, or x^{-3}. As stated earlier, we don't really want to keep negative exponents. We can write this one out to see what really happens to our exponent….

$\underline{x^2} = \underline{\cancel{x} \bullet \cancel{x} \qquad\qquad} = \underline{1}$

x^5 $\cancel{x} \bullet \cancel{x} \bullet x \bullet x \bullet x$ x^3

Make sure that you leave a factor of 1 in the numerator since we cancelled all the factors that were there. As you can see, to make an exponent positive, you simply move the power from the numerator to the denominator and change the sign of the exponent. This also works when moving a power from the denominator to the numerator. You simply "cross the line and change the sign." Be sure to only change the sign of the exponent.

What happens when you are raising a power to a power? Let's take a look.

$(3m^3)^2$

This means that all of $3m^3$ is being squared, or raised to the second power. Therefore each part of the monomial is raised to the power.
Let's write it out so you can see it and then we'll make a statement regarding the rule that applies to a "power to a power."

$3 \bullet m^3 \bullet 3 \bullet m^3 = 9m^6$

We got this answer by using the property above stating that when you have like bases and you're multiplying the powers, you can add the exponents. However, this also leads us to the "power to a power" rule. When raising a power to a power, you can multiply the exponents.
$(a^m)^n = a^{mn}$

There is one more special situation we need to look at. What if something is being raised to the 0 power, like 5^0? To best understand this, let's look a situation that would give us 0 as our exponent.

$$\frac{x^5}{x^5} = x^{5-5} = x^0 = 1$$

You can see here that when we subtract the exponents of our like bases, we get zero. However, you also know that when you have a number divided by itself, you get 1.
$a^0 = 1$